

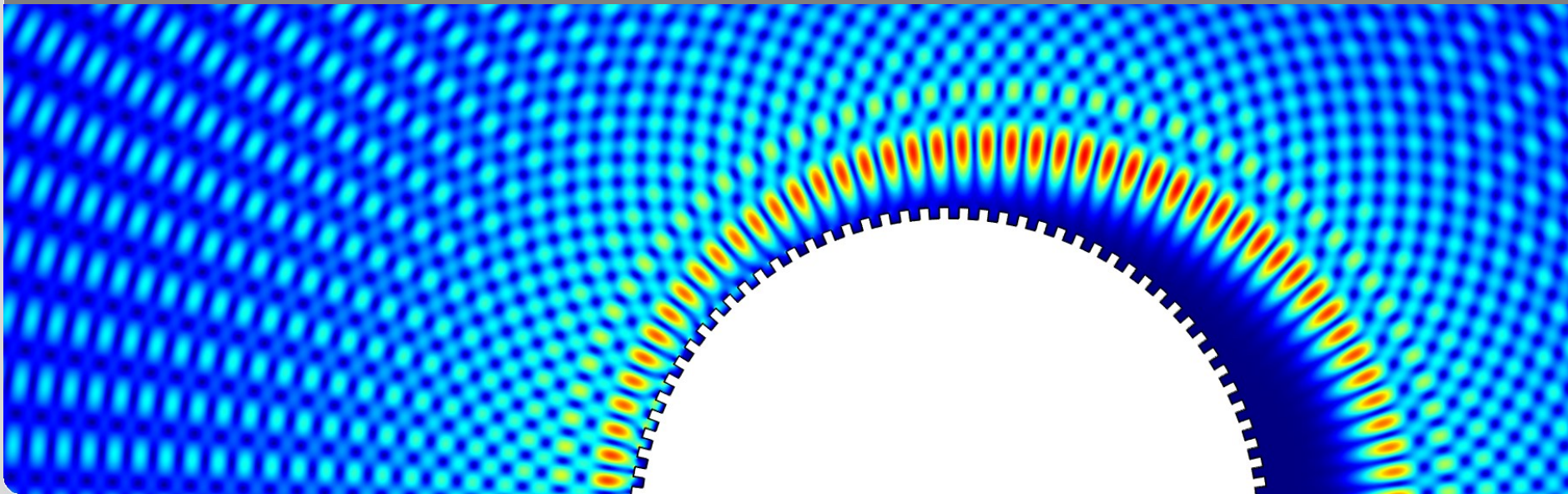
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# Insert Misalignment in Coaxial Gyrotrons: Physical Effects and Numerical Treatment

5<sup>th</sup> ITG International Vacuum Electronics Workshop 2016

Joachim Franck, K. A. Avramidis, G. Gantenbein, S. Illy, I. Gr. Pagonakis, M. Thumm, J. Jelonnek

Institut für Hochleistungsimpuls- und Mikrowellentechnik (IHM)



## ■ Introduction

- Highly Overmoded Gyrotron Cavities
- Coaxial-Cavity Gyrotrons

## ■ Effects and Description of Misalignment

- Mode Content
- Nonuniform Voltage Depression on the Electron Beam

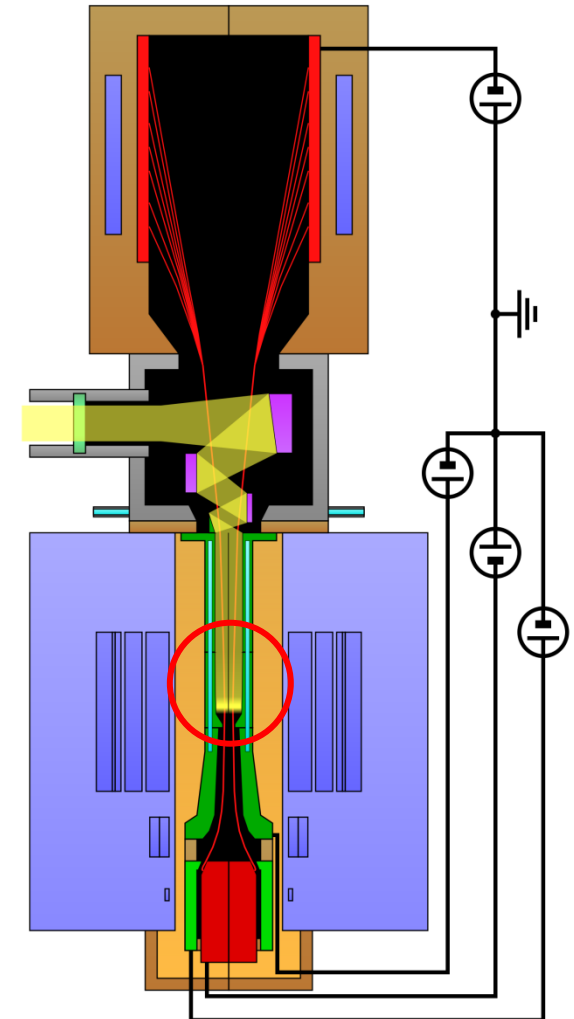
## ■ Conclusion and Outlook

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

# Highly Overmoded Cavities

- Microwave field generated in the gyrotron cavity  
→ ohmic losses on the metallic wall

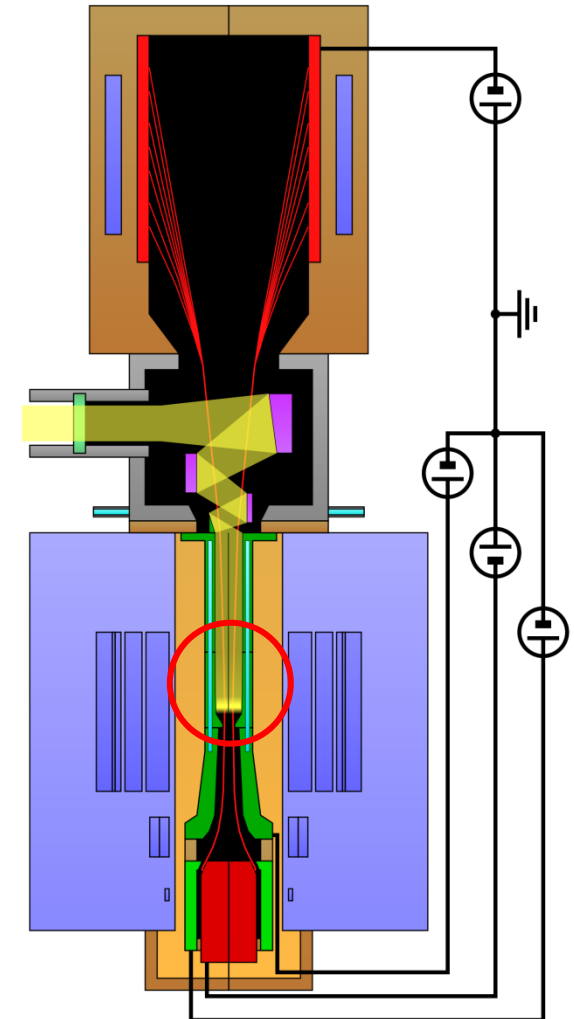
$$P_{\Omega} \approx \frac{2}{\chi_{mp} c} \sqrt{\frac{\pi f}{\mu_0 \sigma}} \cdot \frac{Q_{\text{diff}} P_{\text{out}}}{1 - \left(m/\chi_{mp}\right)^2}$$



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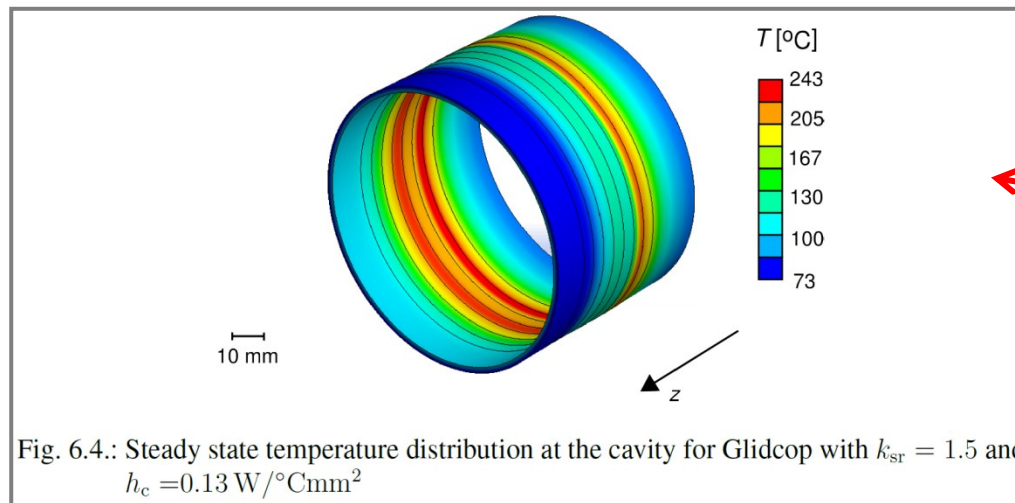


# Highly Overmoded Cavities

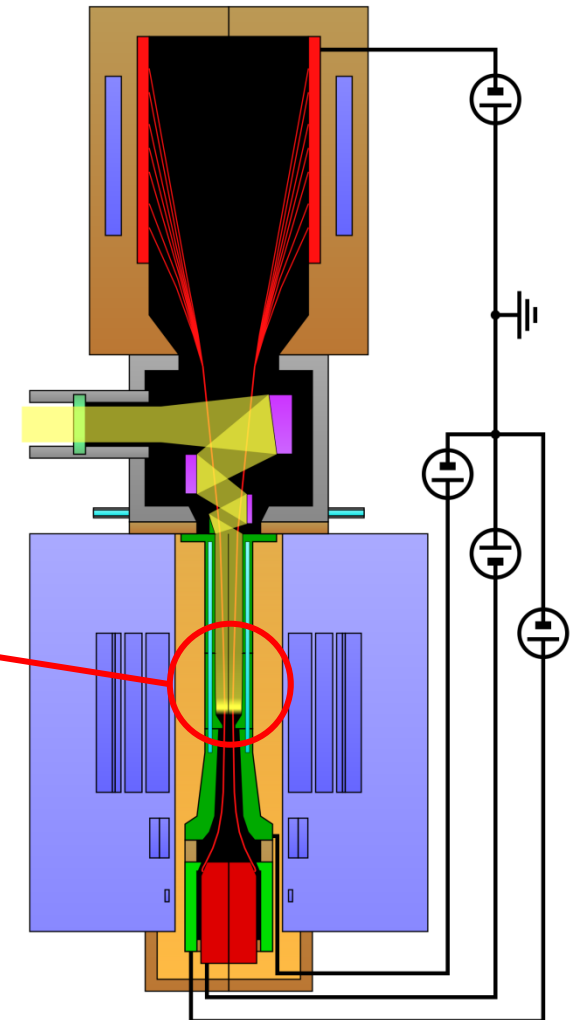
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- Limited cooling capacity per surface area  
(typical maximum loading: 2 kW/cm<sup>2</sup>)  
→ large cavity radius for high power & frequ.



(source: PhD thesis M. Beringer, 2010)

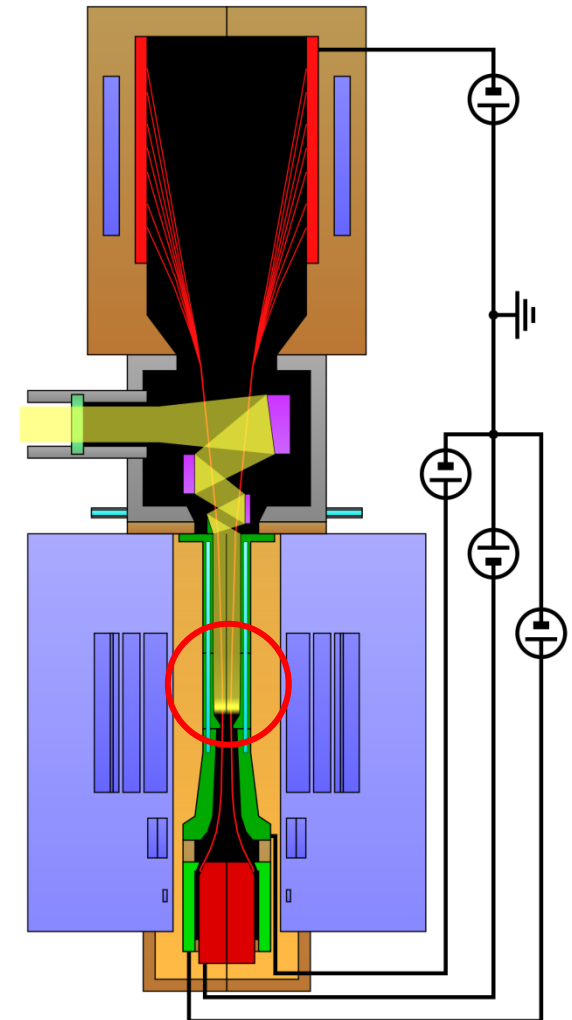


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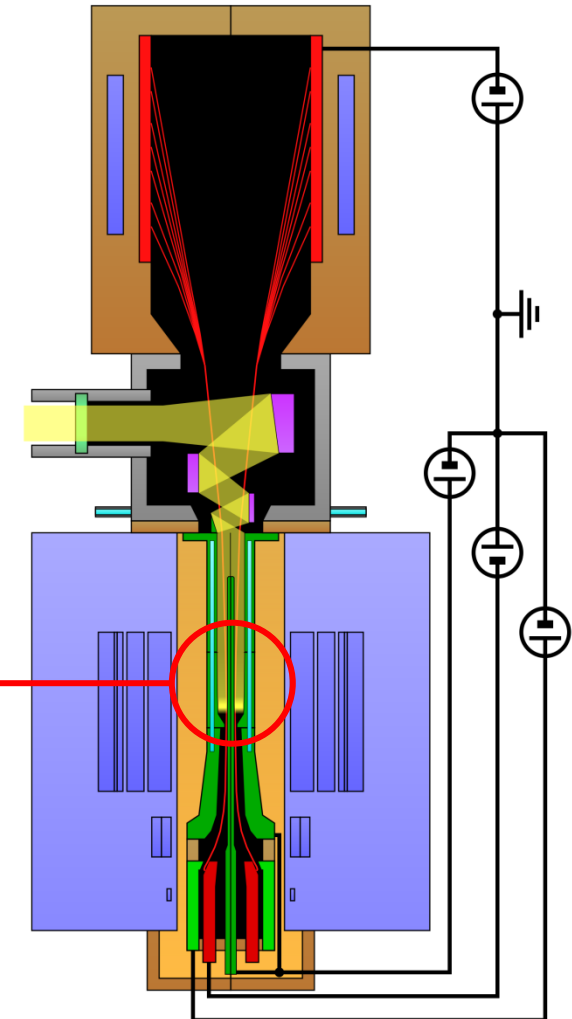
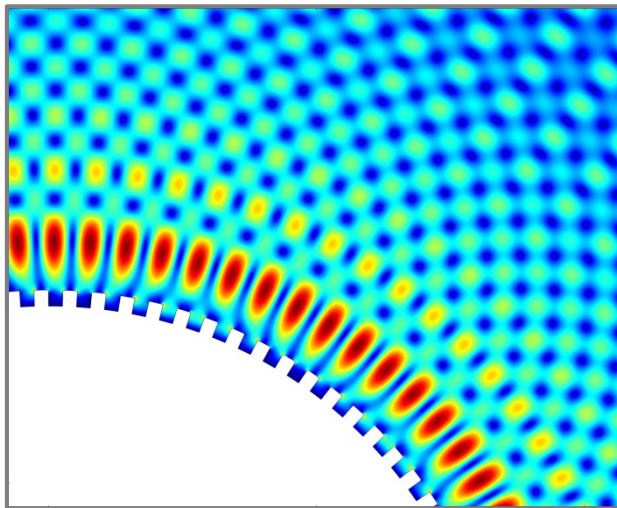
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- Limited cooling capacity per surface area  
(typical maximum loading: 2 kW/cm<sup>2</sup>)  
→ large cavity radius for high power & frequ.
- With  $\omega_{\text{RF}} R_{\text{cav}} = \chi_{mp} c_0$ , this requires high mode eigenvalues  $\chi_{mp}$   
 $m$ : azimuthal index,  $p$ : radial index
- Typical distance between modes:  $\Delta\chi \sim 1.5$   
→ spectral distance  $\Delta\chi/\chi_{mp}$  decreases  
→ mode competition increases  
→ gyrotron operation becomes unstable



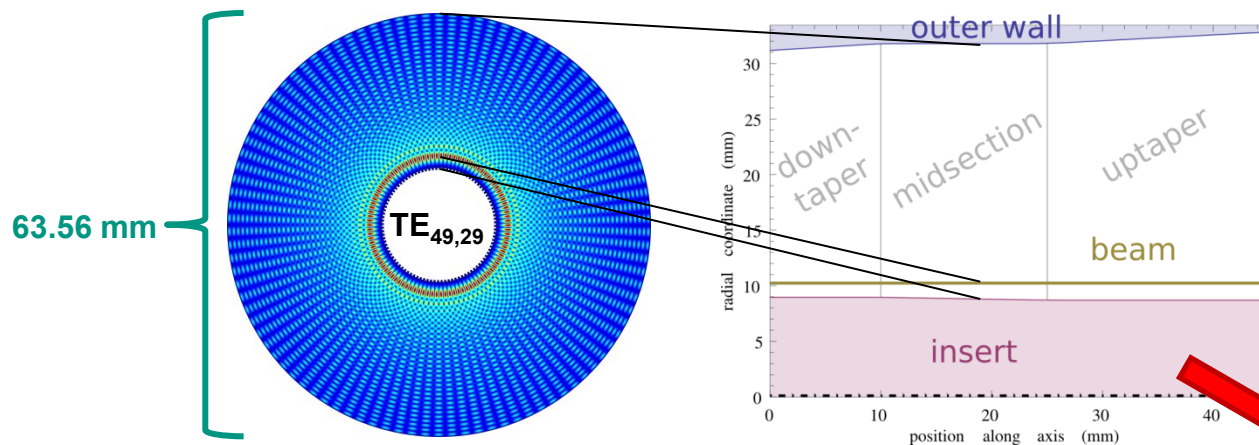
# Coaxial-Cavity Gyrotrons

- (One) solution: coaxial insert
  - Metallic rod (with internal cooling)
  - Fixed in electron gun region of the gyrotron
  - In the cavity region: longitudinally corrugated & tapered
- Insert interacts with some modes
  - rarefaction of spectrum possible
  - less mode competition
  - more stable operation

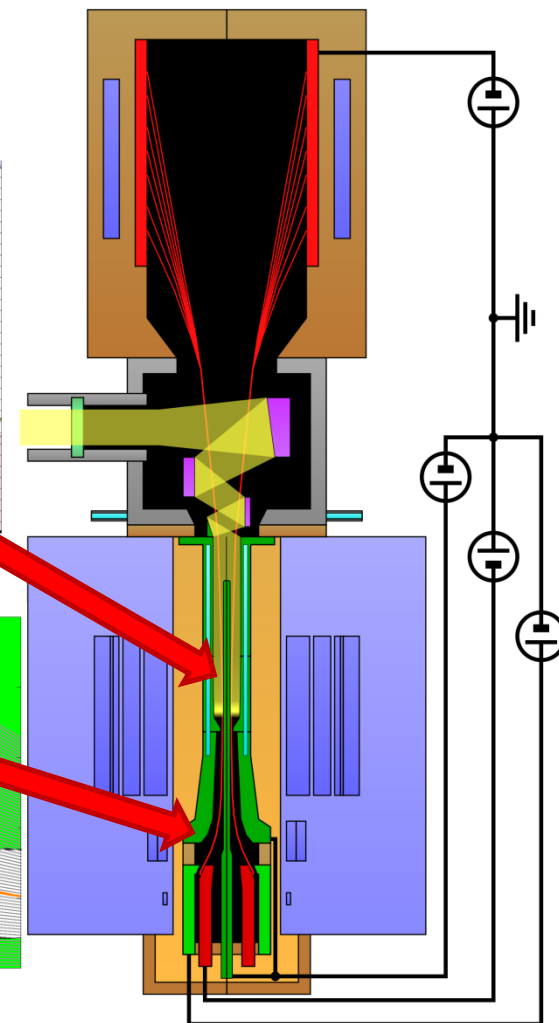
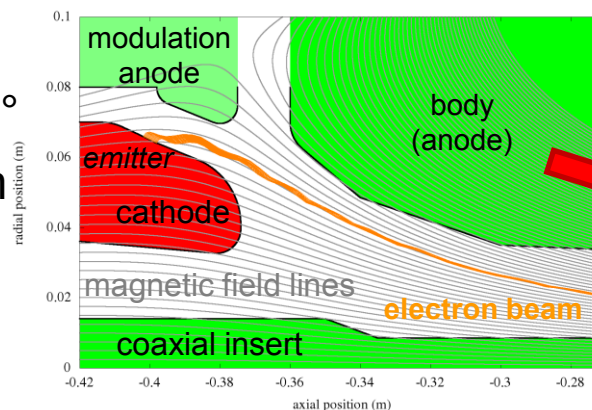


# Coaxial-Cavity Gyrotrons: The 238 GHz Design

- coaxial-cavity  $TE_{49,29}$  ( $\chi_{49,29} \approx 158$ ) @ 237.5 GHz: **1.9 MW** (33 %)
- $R_O=31.78$  mm,  $R_I=8.66$ -8.40 mm (taper angle:  $-1^\circ$ ),  $r_B=10.24$  mm
- 100 corrugations with depth  $D_{\text{corr}} = 0.3$  mm
- $I_B=69.3$  A;  $U_B=85.6$  kV;  $U_{\text{mod}} \approx 36$  kV ( $\alpha=1.22$ );  $B=9.58$  T



- $r_{\text{MIG}}=80$  mm
- $r_E=65$  mm,  $d_E=4.3$  mm,  $\theta_E=25^\circ$
- emtr. surface roughness  $2 \mu\text{m}$
- no reflected electrons
- reduced potential wells
- max. electric field: 7 kV/mm



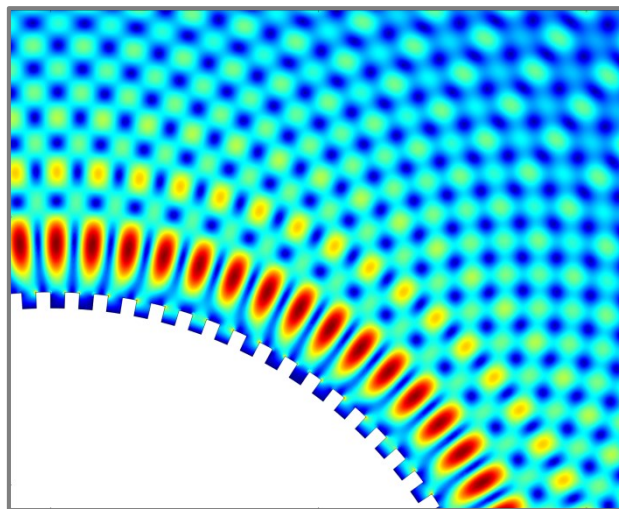
# Coaxial-Cavity Gyrotrons: The 238 GHz Design

## ■ Technical issue:

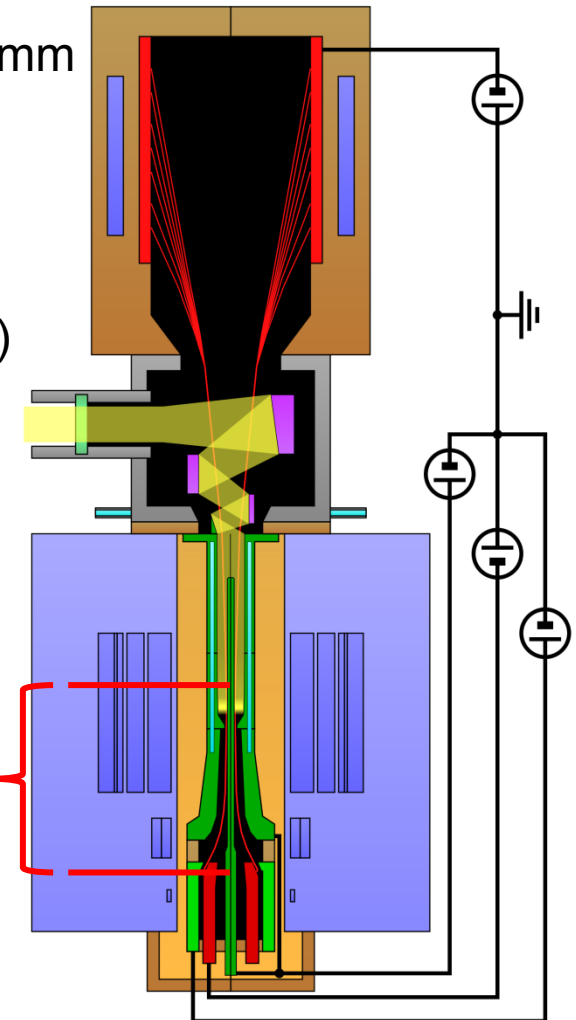
- insert length  $\approx 50$  cm
- typ. dimensions in cavity:  $\lambda = 1.26$  mm,  $D_{\text{corr}} = 0.3$  mm  
→ required tilt:  $< 0.1^\circ$

## ■ Tubes are welded and evacuated, thus:

- Possible misalignment cannot be undone
- Only freedom: electron beam position (magn. field)



5 mm  
~50 cm



Question to design specifications:

**What is the maximum tolerable insert misalignment?**

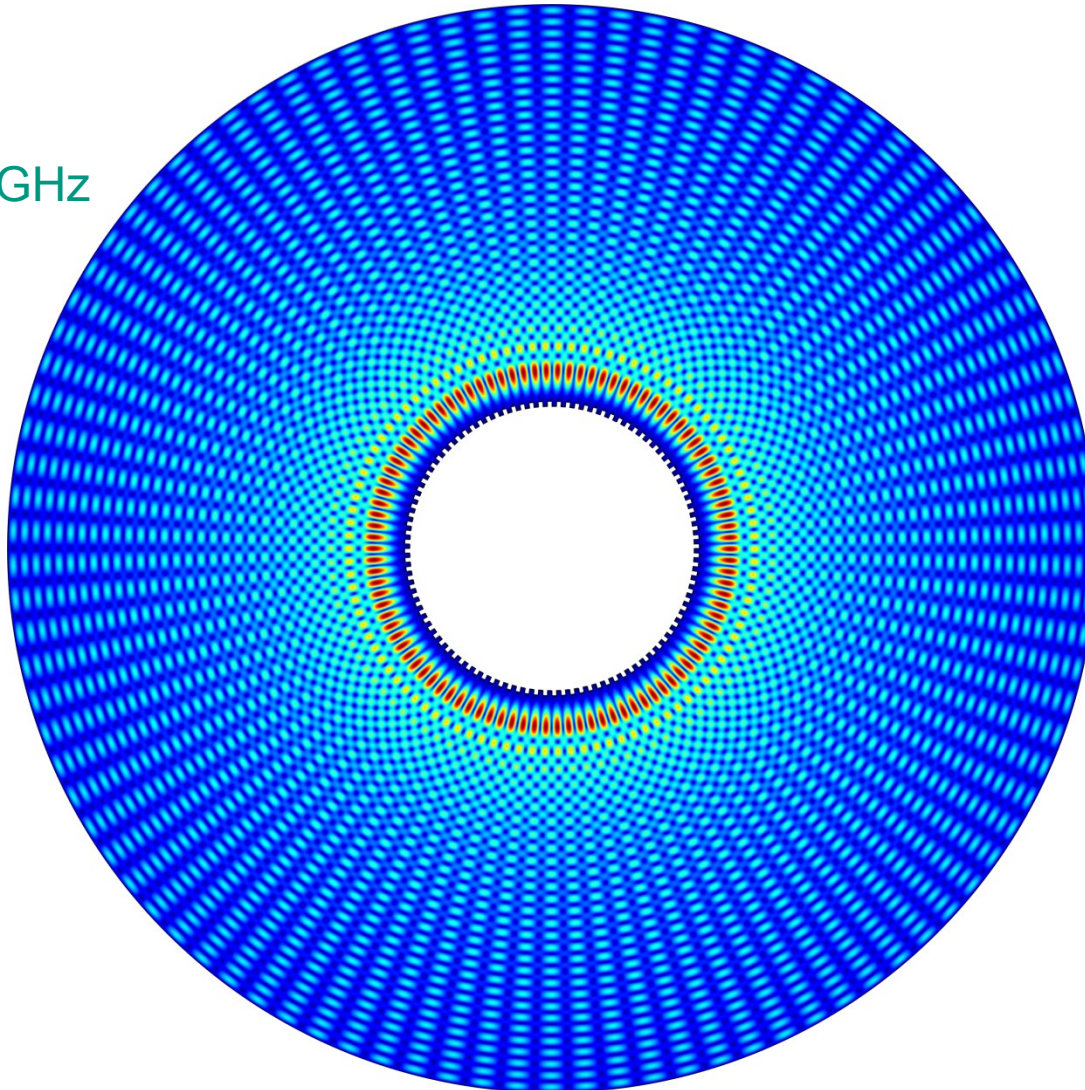
Two aspects of insert misalignment:

- **Influence on the electromagnetic modes**
- **Influence on the electron beam**

# Effects of Misalignment 1: Mode Content

■ Mode TE<sub>49,29</sub> at 0.0 mm misalignment

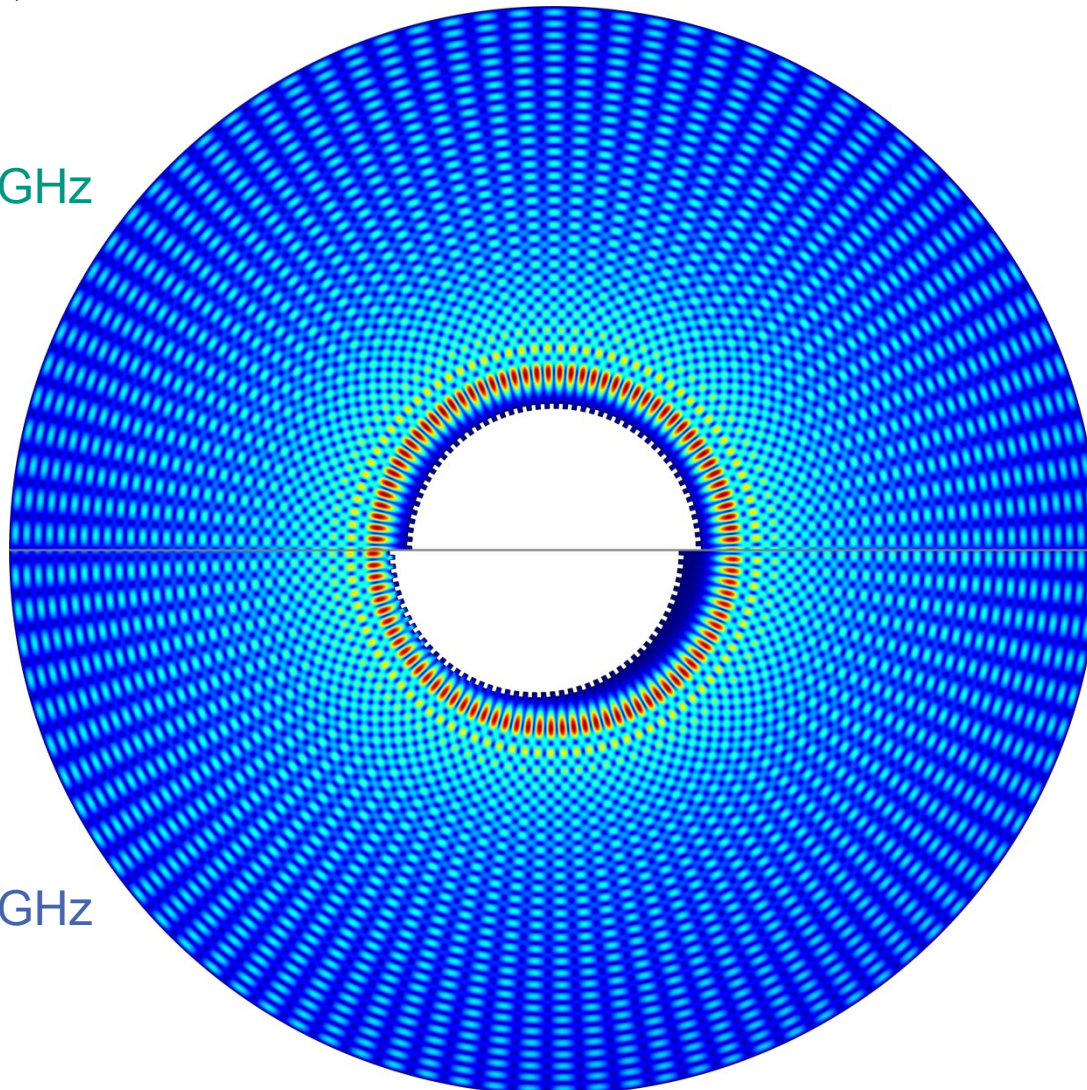
$f_{\text{cutoff}} = 237.31 \text{ GHz}$



# Effects of Misalignment 1: Mode Content

■ Mode TE<sub>49,29</sub> at 0.0 mm / 1.0 mm misalignment

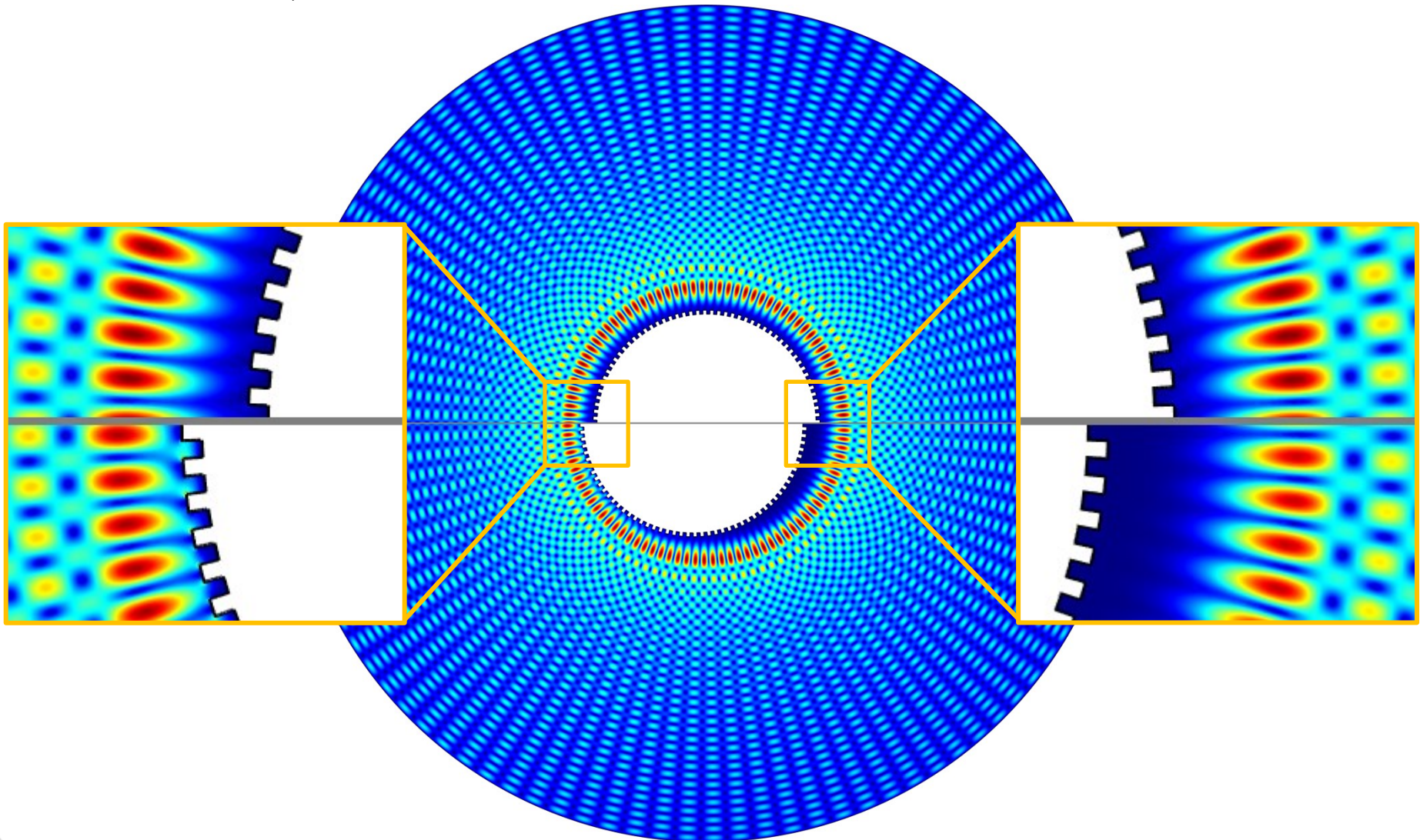
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$f_{\text{cutoff}} = 237.37 \text{ GHz}$

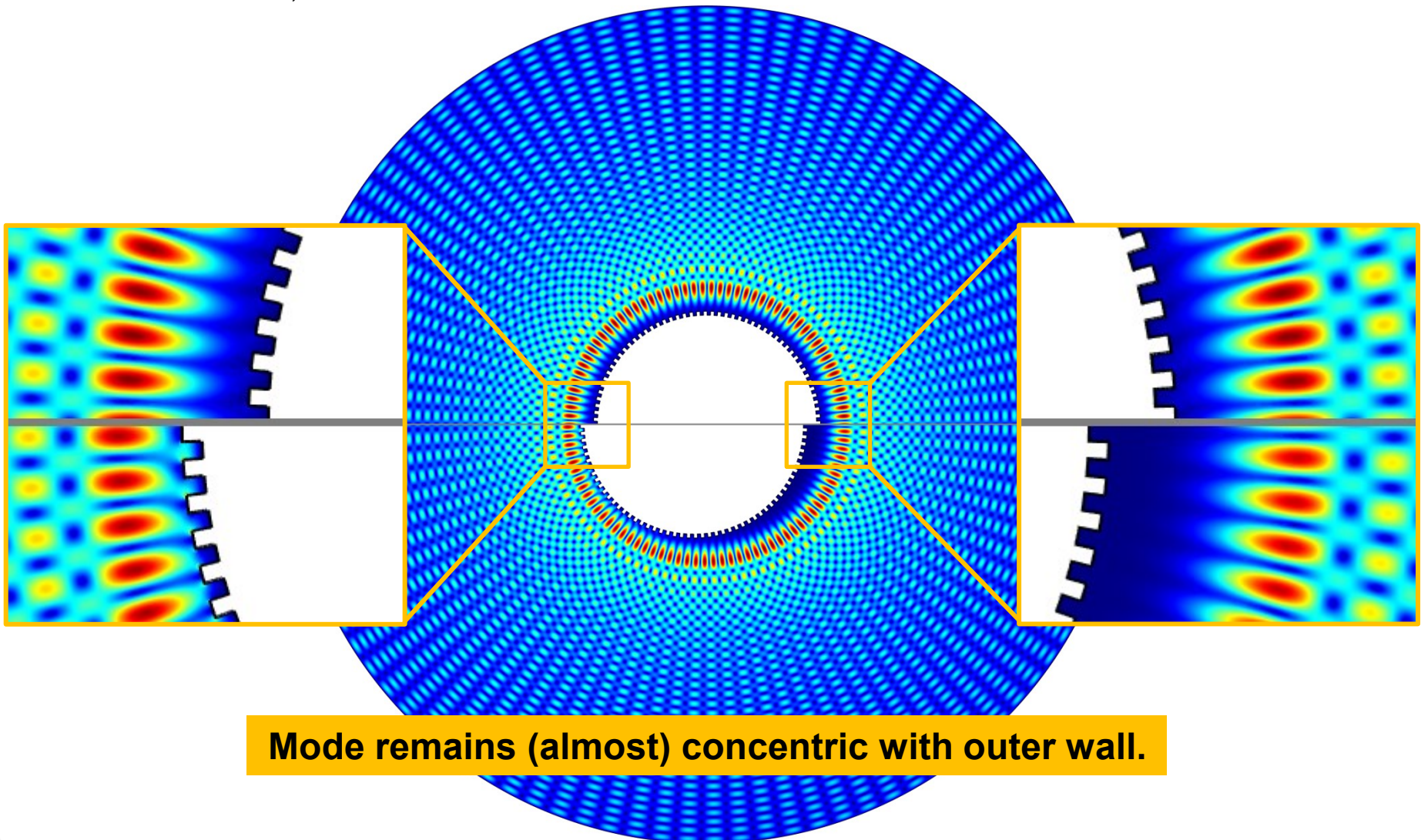
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■ Mode  $TE_{49,29}$  at 0.0 mm / 1.0 mm misalignment



# Effects of Misalignment 1: Mode Content

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# Description of Misalignment: Mode Content

Small, but important changes in deformed modes, e.g.:

- Mode eigenvalue  $\chi_{mp}$  ( $\rightarrow$  frequency; required precision:  $\sim 10^{-4}$  (1:1.000.000))
- Field distribution  $\rightarrow$  coupling to electron beam (required precision:  $\sim 10^{-2}$ )

Goal: analytic description of TE modes which works for

- Corrugated insert (Surface Impedance Model, i.e. infinitely small corrugations)
- Misalignment up to **1 mm**
- Eigenvalues up to **180**
- Eigenvalue precision of  **$10^{-5}$**
- Azimuthal indices  $m$  up to **60**

and (as a code) is suitable as input for interaction codes.

# Description of Misalignment: Mode Content

■ Field decomposition  $\hat{B}_{z,m}(r, \varphi) \sim \sum_{q=-\infty}^{\infty} K_{mq}(s, r, r', C, \delta) B_q e^{-iq\varphi}$

■ Modes: solutions of  $\hat{B}'_{z,m}(R_0) = 0$

$$K_{mq}(s, r, r', C, \delta) = \overbrace{J_{q-m}(\delta)}^{\text{misalignment}} \cdot \underbrace{(J_{m-s}(k_{\perp}r) \quad -N_{m-s}(k_{\perp}r))}_{\text{wall boundary condition}} \cdot \overbrace{\begin{pmatrix} N'_q(k_{\perp}r') & N_q(k_{\perp}r') \\ J'_q(k_{\perp}r') & J_q(k_{\perp}r') \end{pmatrix}}^{\text{coaxial insert boundary condition}} \cdot \underbrace{\begin{pmatrix} 1 \\ C \end{pmatrix}}_{\text{corrugations}}$$

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■ “Small-matrix approach”

vs.

“Large-matrix approach”

$$K'_{m_0, \Delta m} = \begin{pmatrix} K'_{m_0 - \Delta m, m_0 - \Delta m} & \cdots & K'_{m_0 - \Delta m, m_0 + \Delta m} \\ \vdots & K'_{m_0, m_0} & \vdots \\ K'_{m_0 + \Delta m, m_0 - \Delta m} & \cdots & K'_{m_0 + \Delta m, m_0 + \Delta m} \end{pmatrix}$$

$$K'_{m_{\max}, \Delta m} = \begin{pmatrix} K'_{0, \Delta m} & & & \\ & \ddots & & \\ & & K'_{m_{\max}, \Delta m} & \\ & & & \ddots \end{pmatrix}$$

$$K_{mq}(s, r, r', C, \delta) = J_{q-m}(\delta) \cdot (J_{m-s}(k_{\perp} r) - N_{m-s}(k_{\perp} r)) \cdot \begin{pmatrix} N'_q(k_{\perp} r') & N_q(k_{\perp} r') \\ J'_q(k_{\perp} r') & J_q(k_{\perp} r') \end{pmatrix} \cdot \begin{pmatrix} 1 \\ C \end{pmatrix}$$

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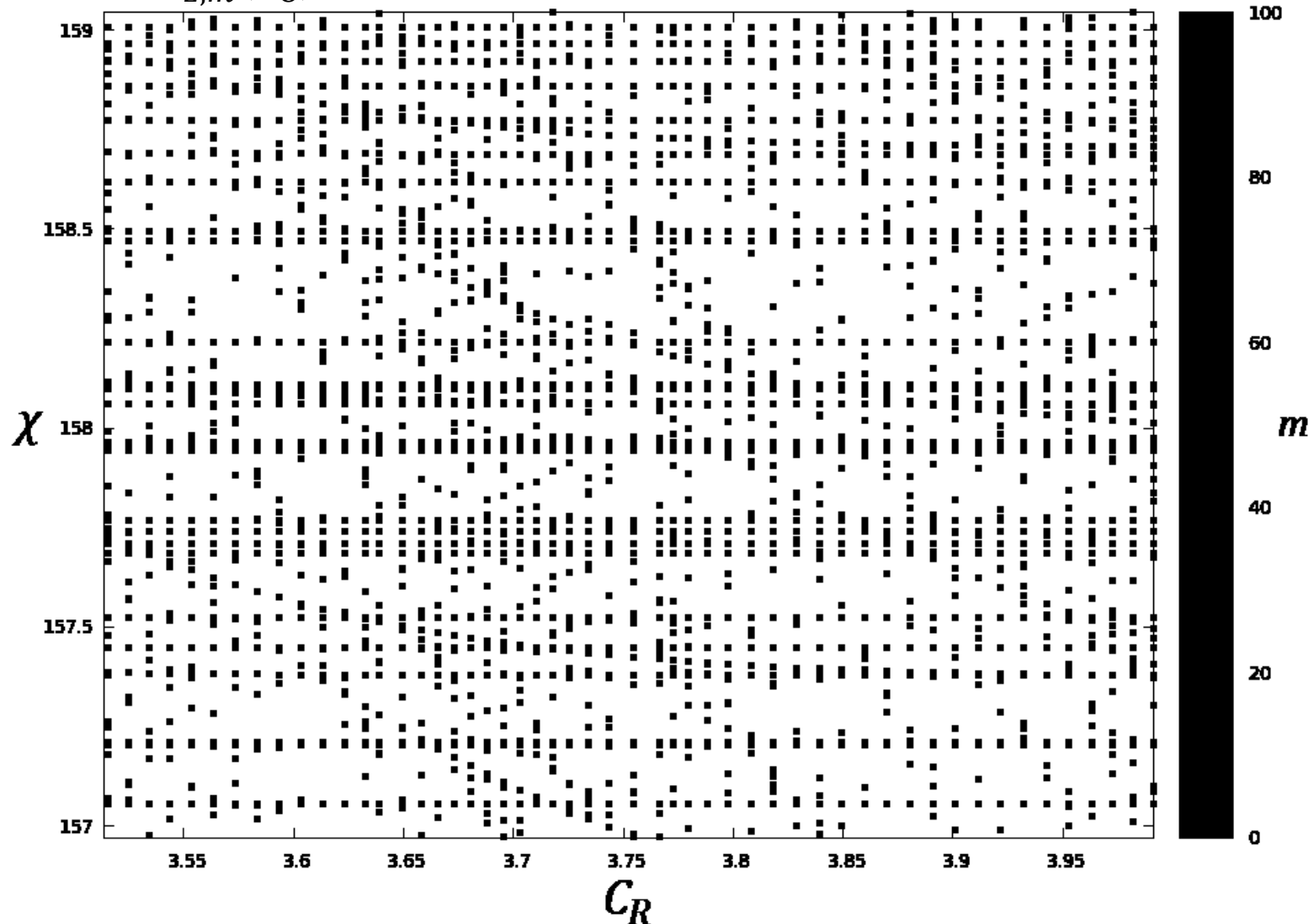
vs. “Large-matrix approach”

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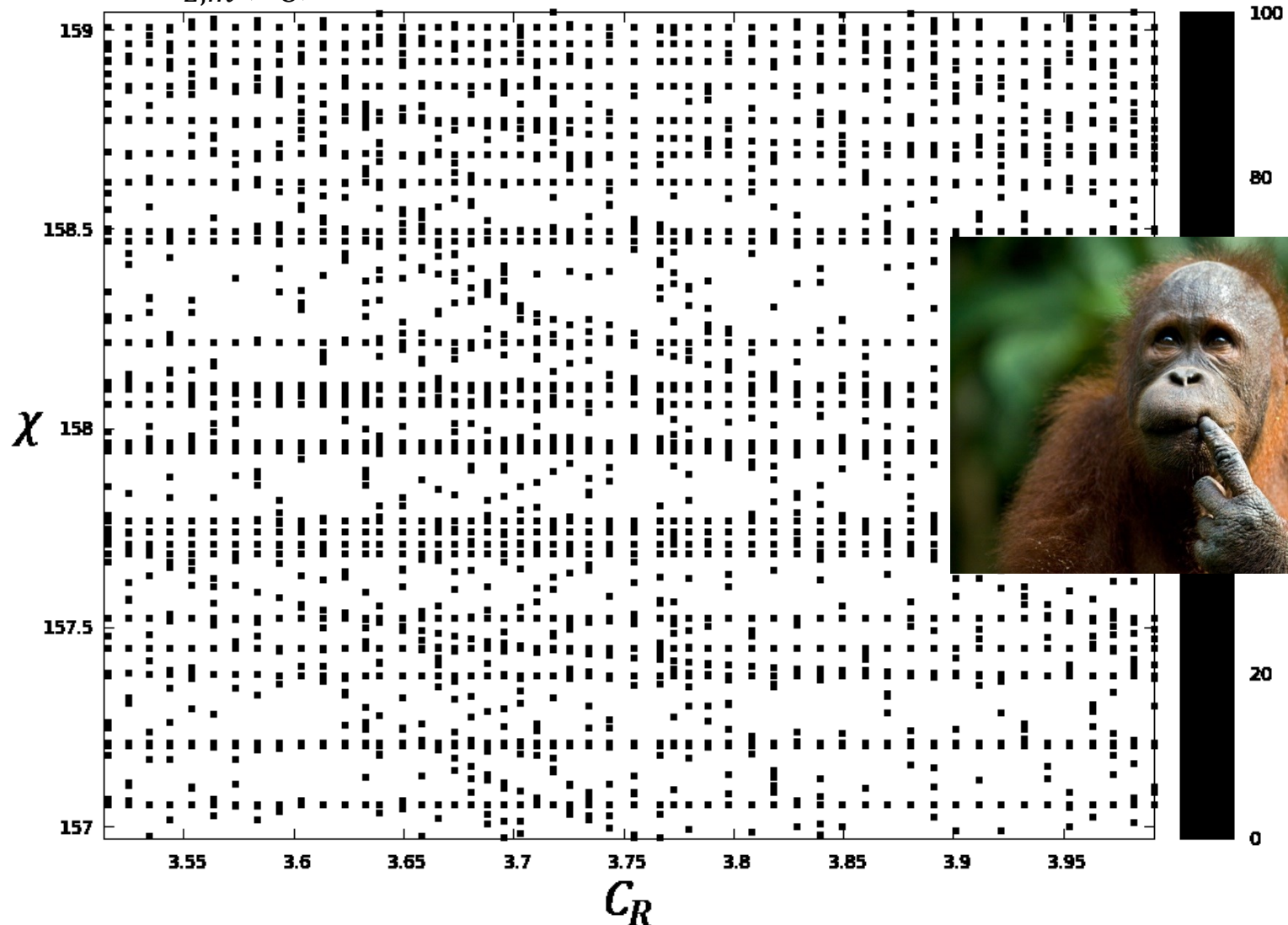
# Description of Misalignment: Mode Content

Result of  $\hat{B}'_{z,m}(R_0) = 0$



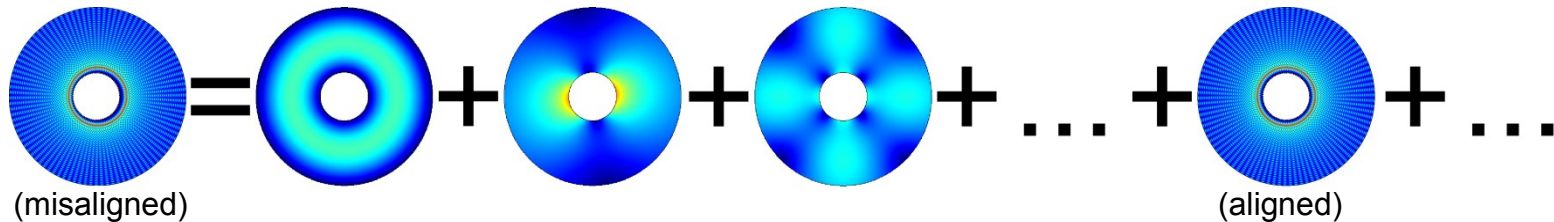
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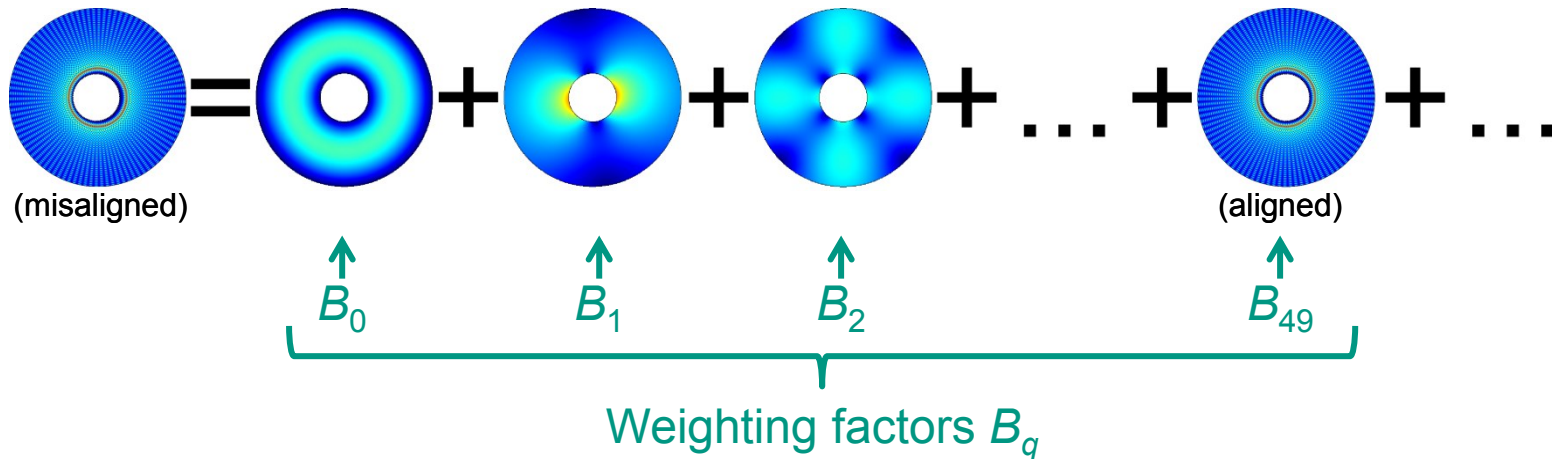
- Field decomposition  $\hat{B}_{z,m}(r, \varphi) \sim \sum_{q=-\infty}^{\infty} K_{mq}(s, r, r', C, \delta) B_q e^{-iq\varphi}$
- e.g.  $m=49$ :



$$K_{mq}(s, r, r', C, \delta) = J_{q-m}(\delta) \cdot (J_{m-s}(k_{\perp}r) - N_{m-s}(k_{\perp}r)) \cdot \begin{pmatrix} N'_q(k_{\perp}r') & N_q(k_{\perp}r') \\ J'_q(k_{\perp}r') & J_q(k_{\perp}r') \end{pmatrix} \cdot \begin{pmatrix} 1 \\ C \end{pmatrix}$$

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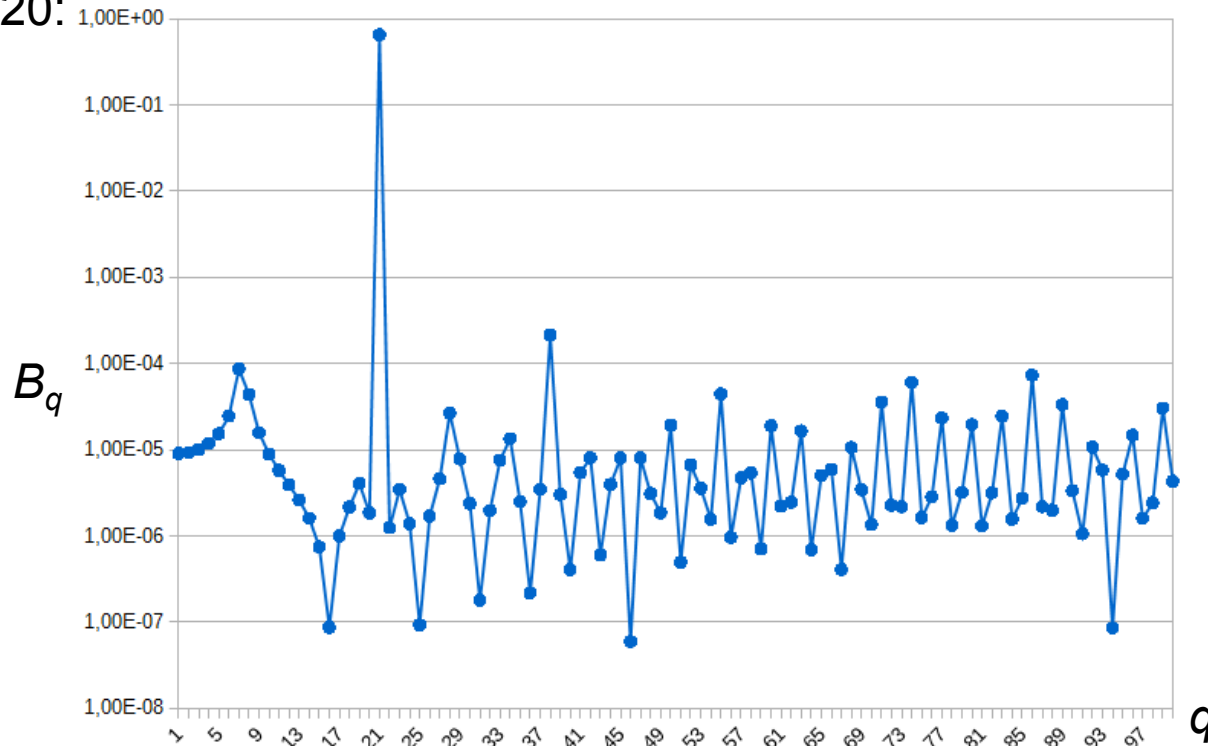


$$K_{mq}(s, r, r', C, \delta) = J_{q-m}(\delta) \cdot (J_{m-s}(k_{\perp}r) - N_{m-s}(k_{\perp}r)) \cdot \begin{pmatrix} N'_q(k_{\perp}r') & N_q(k_{\perp}r') \\ J'_q(k_{\perp}r') & J_q(k_{\perp}r') \end{pmatrix} \cdot \begin{pmatrix} 1 \\ C \end{pmatrix}$$

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e.g.  $m=20$ :

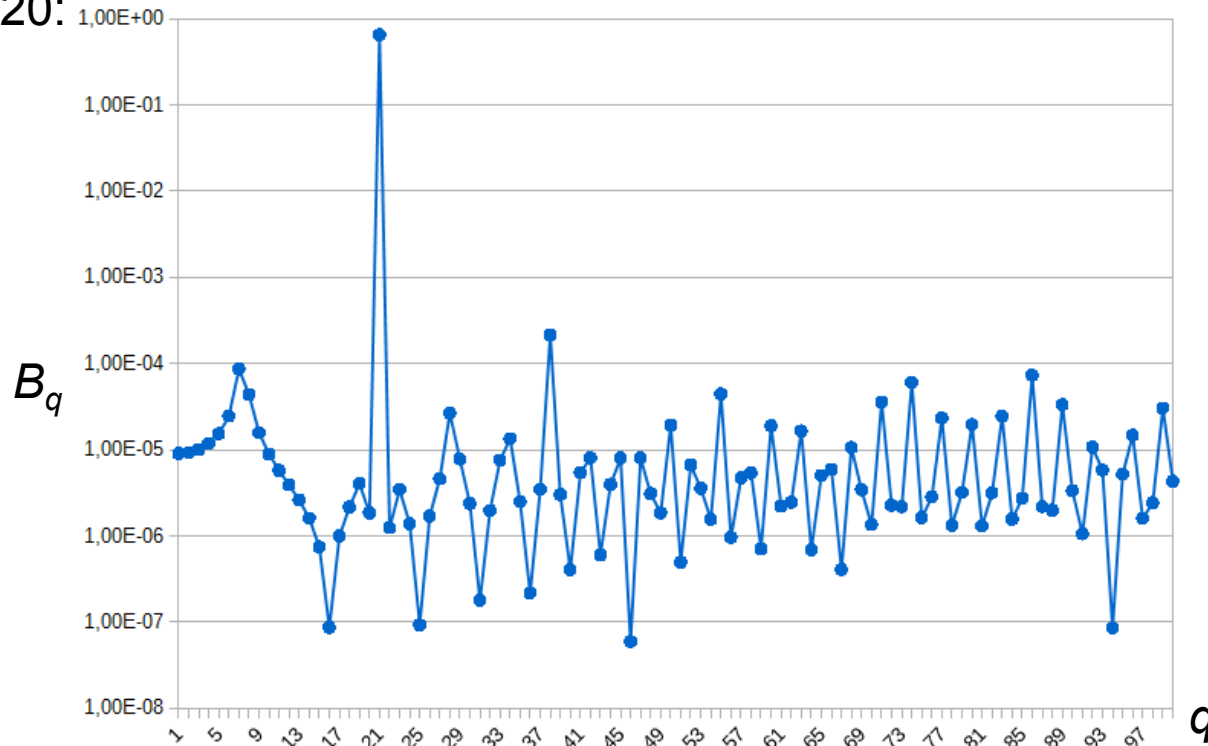


- Index  $q$  of largest weighting factor is used as **azimuthal index  $m$** .  
(largest weighting factor  $\sim$  largest energy content)

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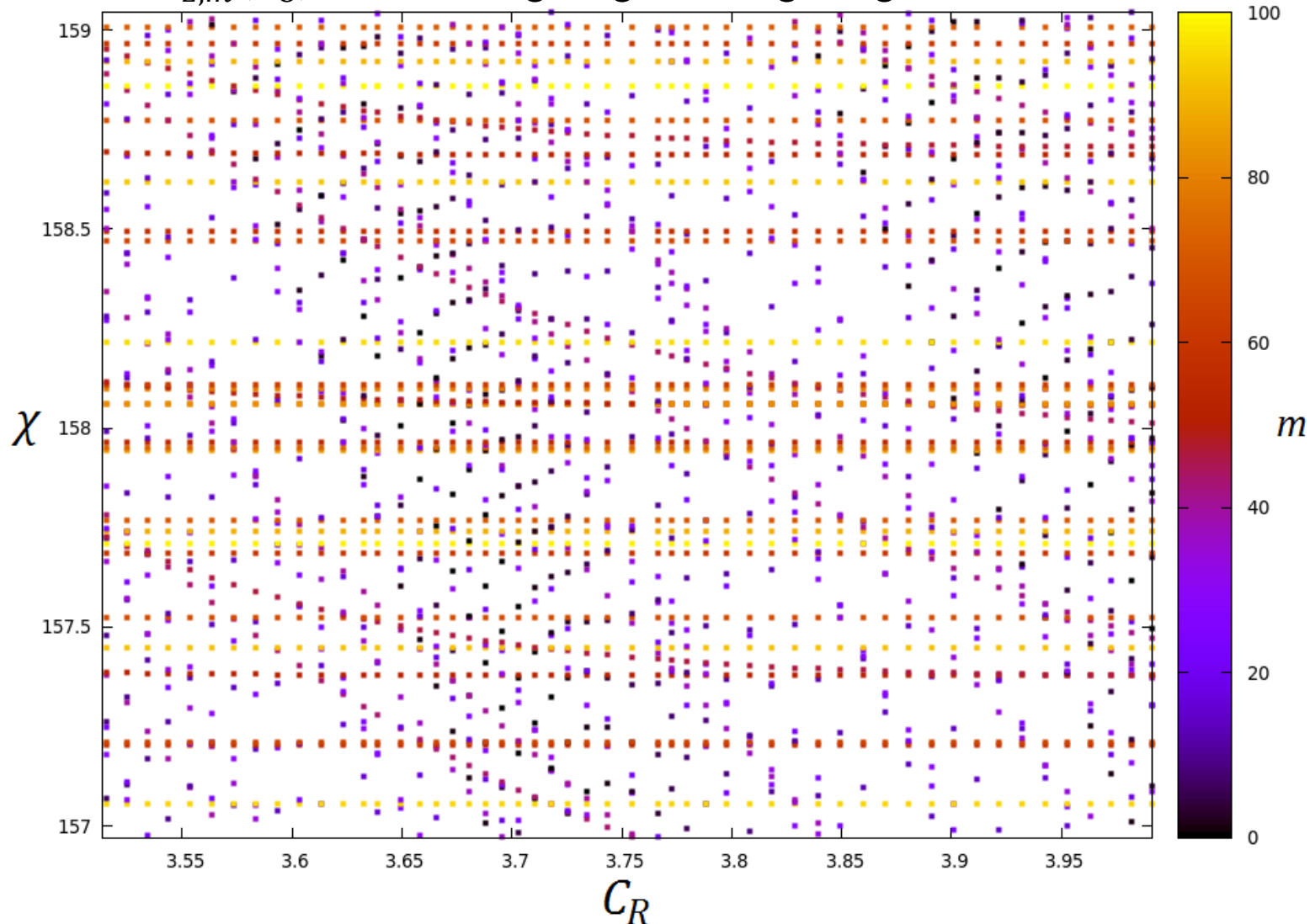
e.g.  $m=20$ :



- Index  $q$  of largest weighting factor is used as **azimuthal index  $m$** .
- Number  $p$  of root  $\chi_{mp, \text{misaligned}} = k_{\perp} R_O$  is used as **radial index  $p$** .

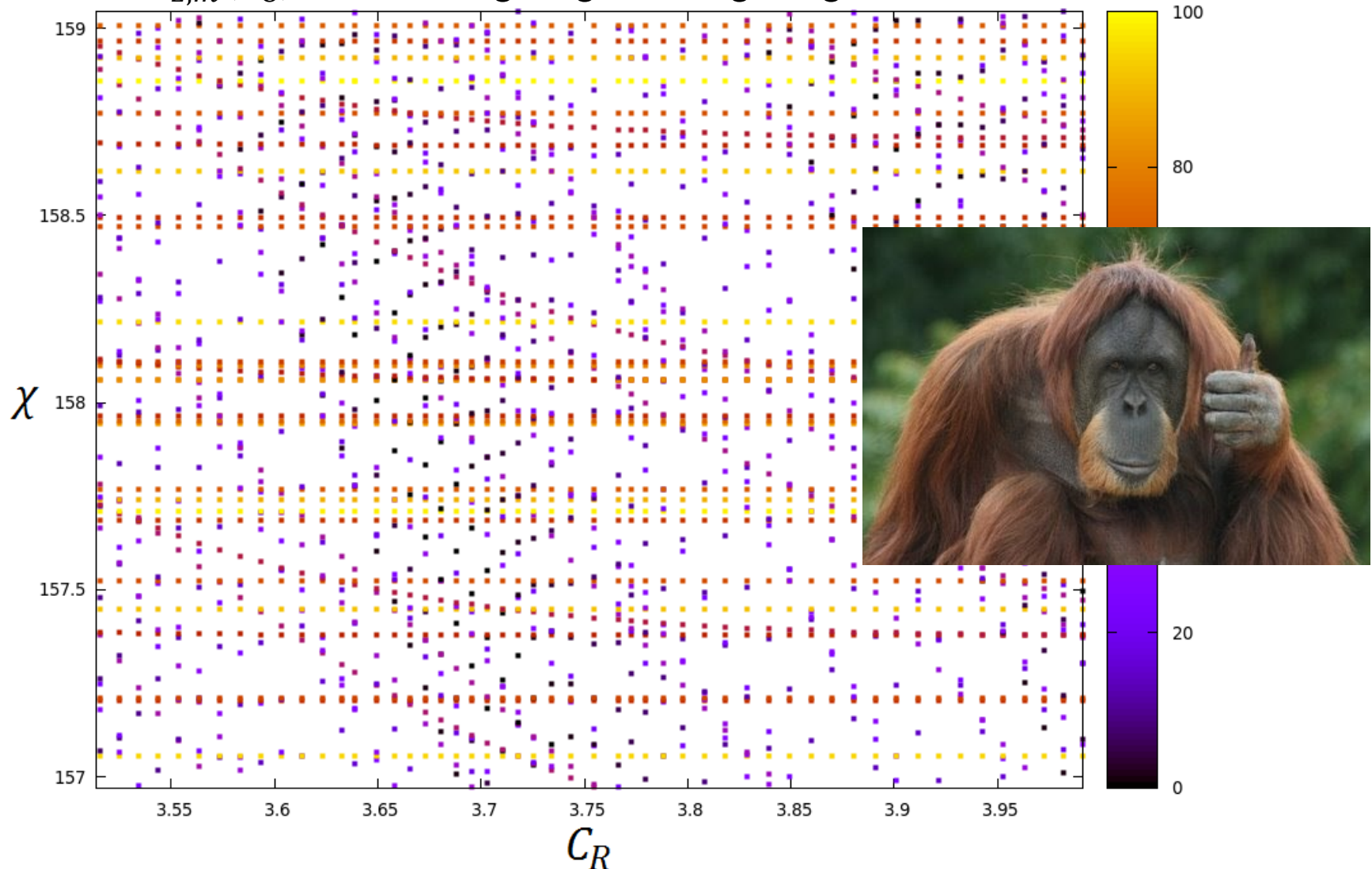
# Description of Misalignment: Mode Content

Result of  $\hat{B}'_{z,m}(R_0) = 0$  , using largest weighting factor



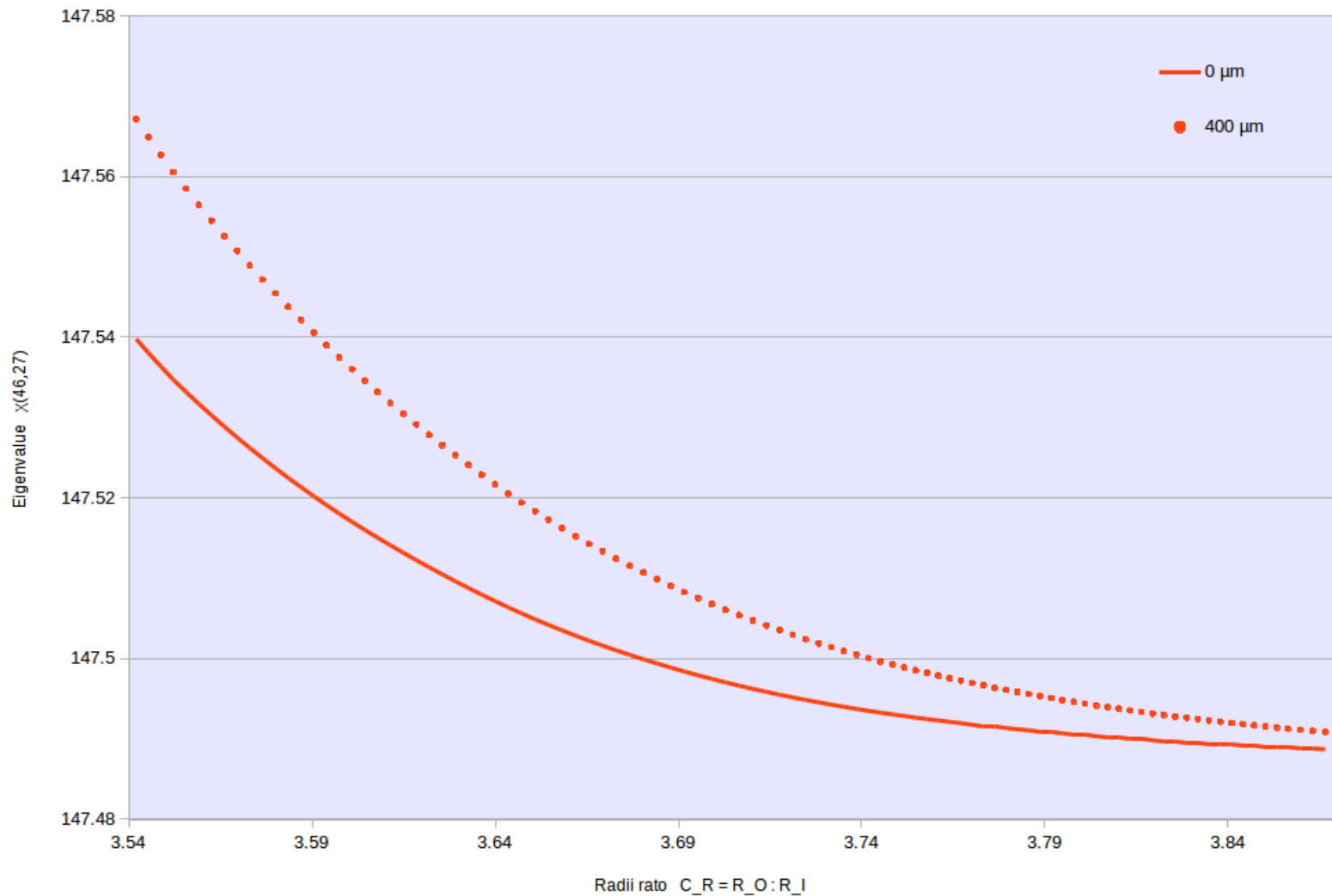
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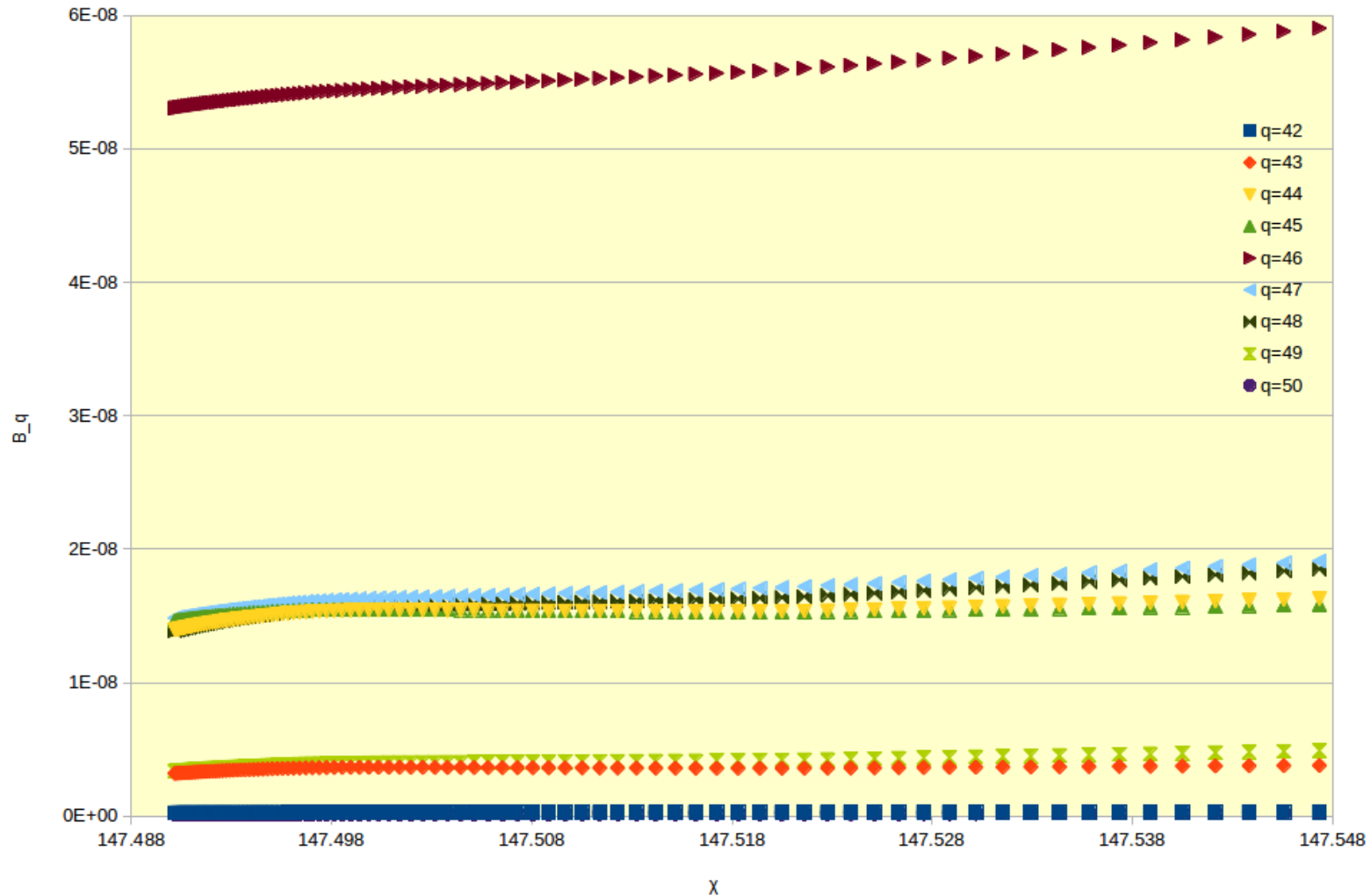
# Description of Misalignment: Mode Content

TE<sub>46,27</sub>: Eigenvalue versus misalignment



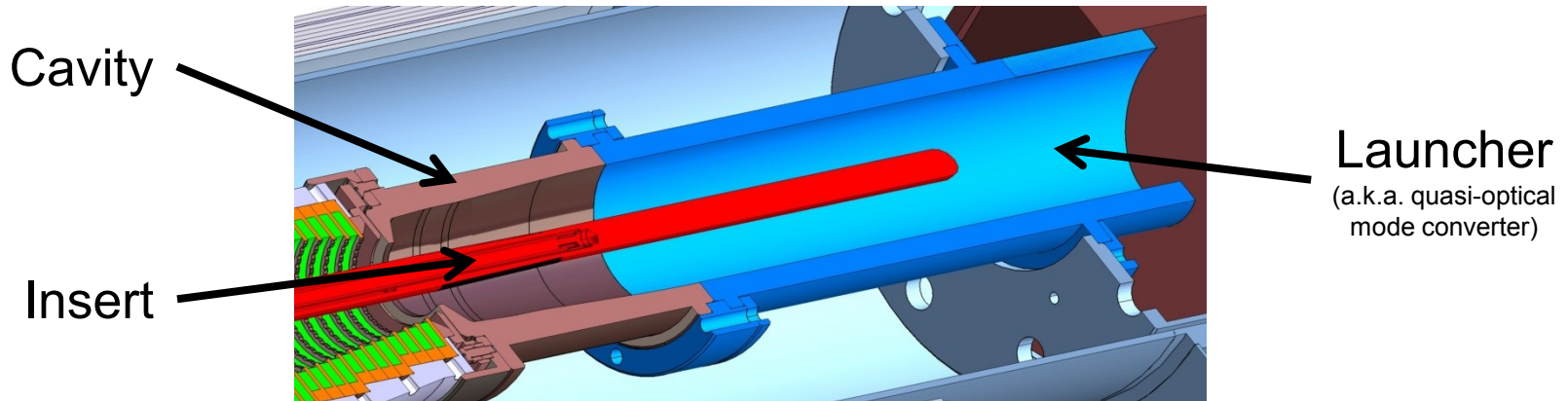
# Description of Misalignment: Mode Content

TE<sub>46,27</sub> (300 μm): Azimuthal components versus eigenvalue



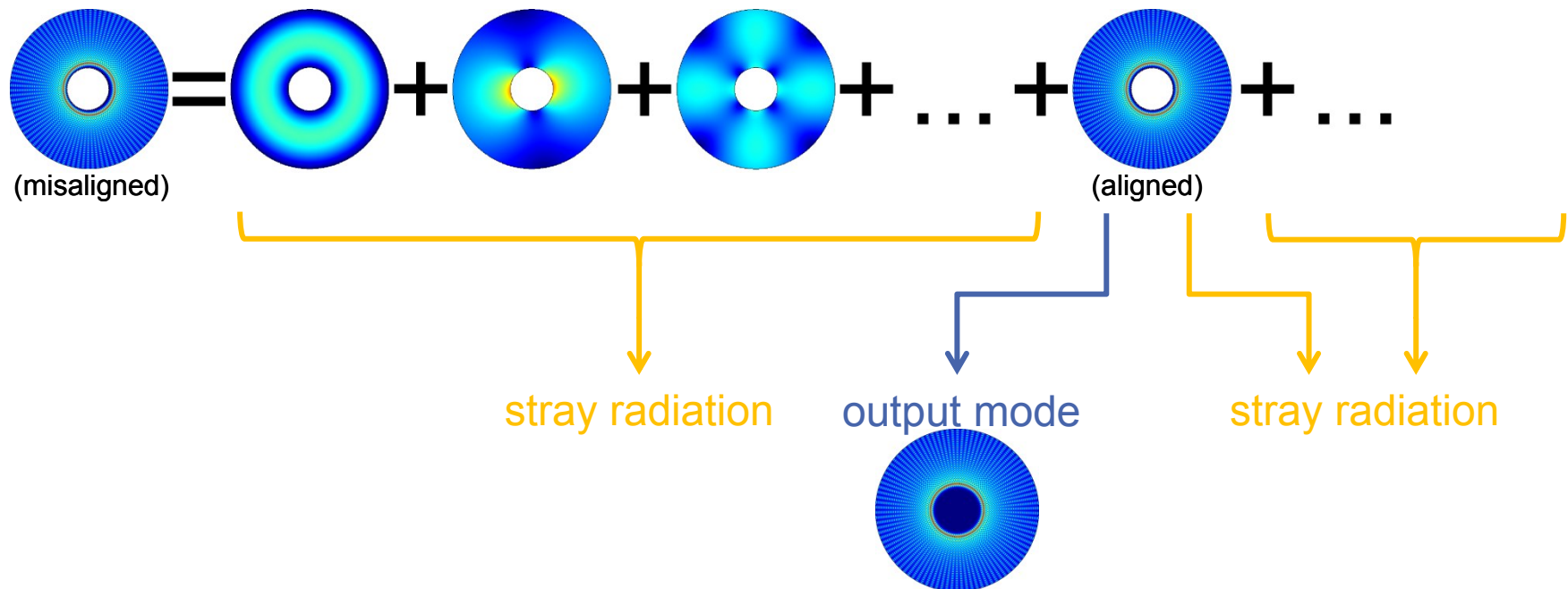
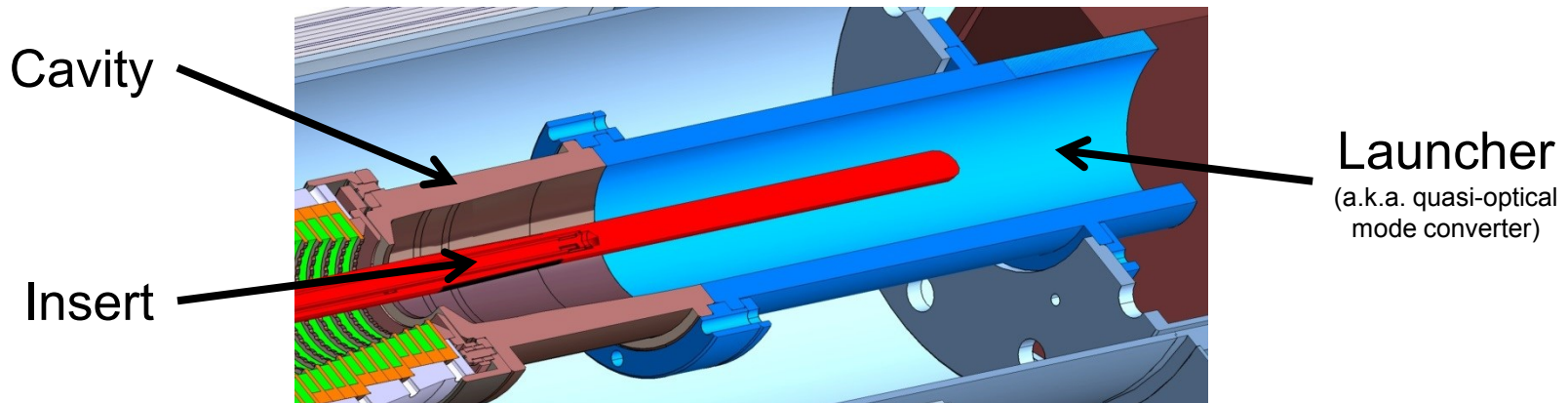
# Description of Misalignment: Mode Content

- (mis)aligned insert ends after cavity → mode conversion



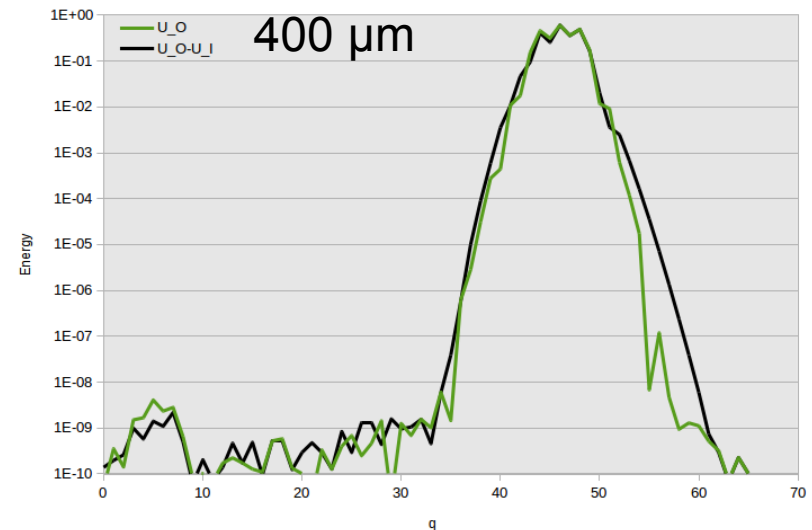
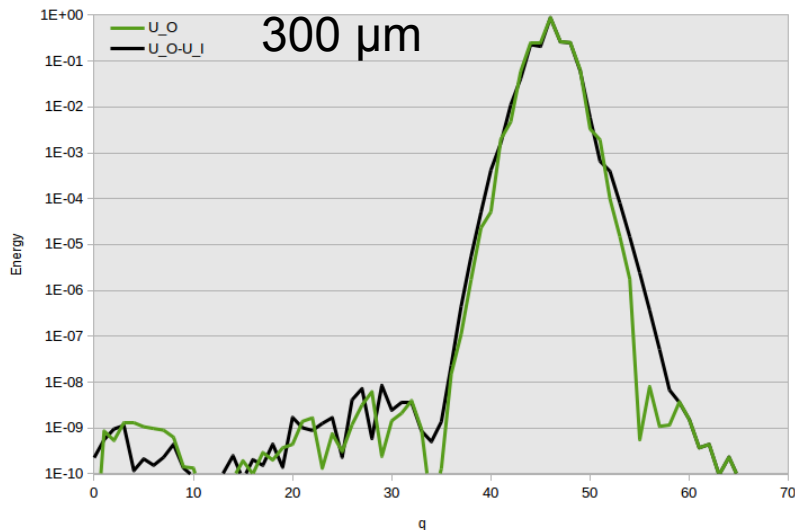
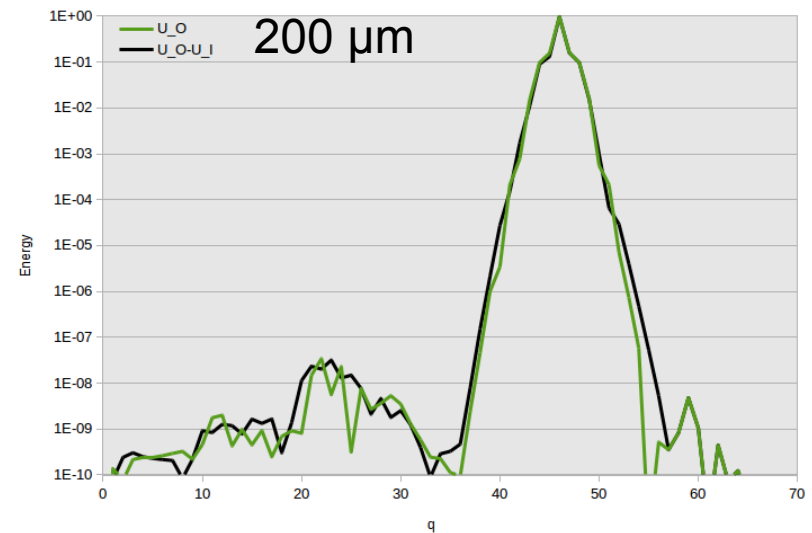
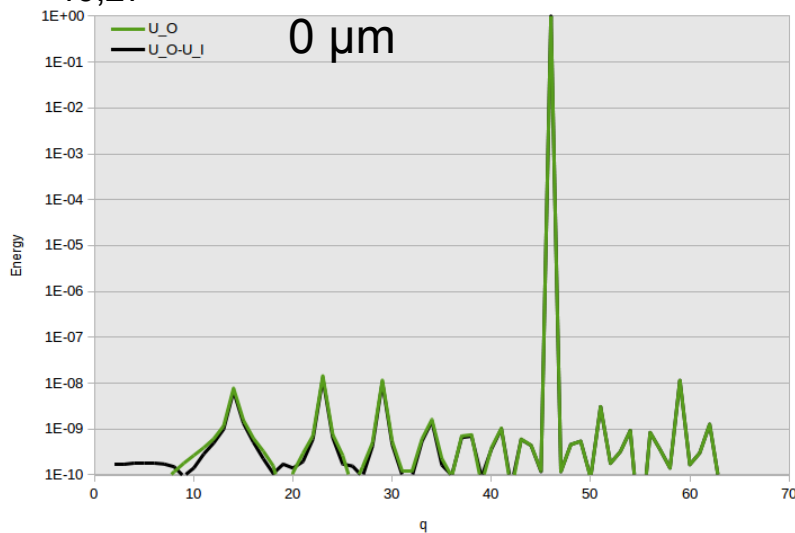
# Description of Misalignment: Mode Content

- (mis)aligned insert ends after cavity → mode conversion



# Description of Misalignment: Mode Content

TE<sub>46,27</sub>: Energy content versus misalignment (→ stray radiation)



# Description of Misalignment: Mode Content

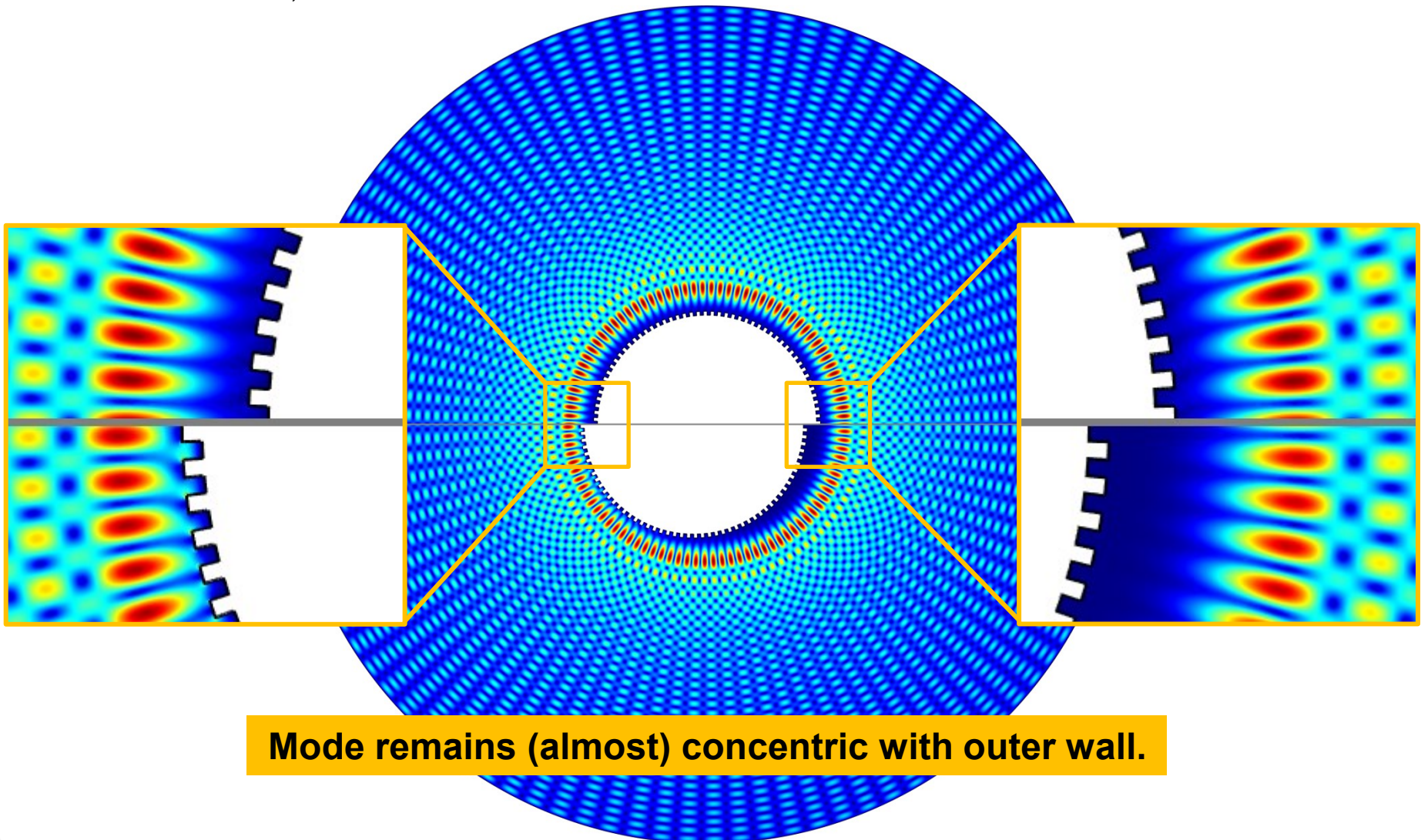
Procedure result:

- Every mode in cavities with (reasonably) misaligned insert has a label  $TE_{m,p}$ .
- Proper labelling ensures proper calculation of
  - axial mode profile
  - spurious mode content
  - ohmic wall loading
  - coupling to the electron beam

→ suitable for interaction simulations

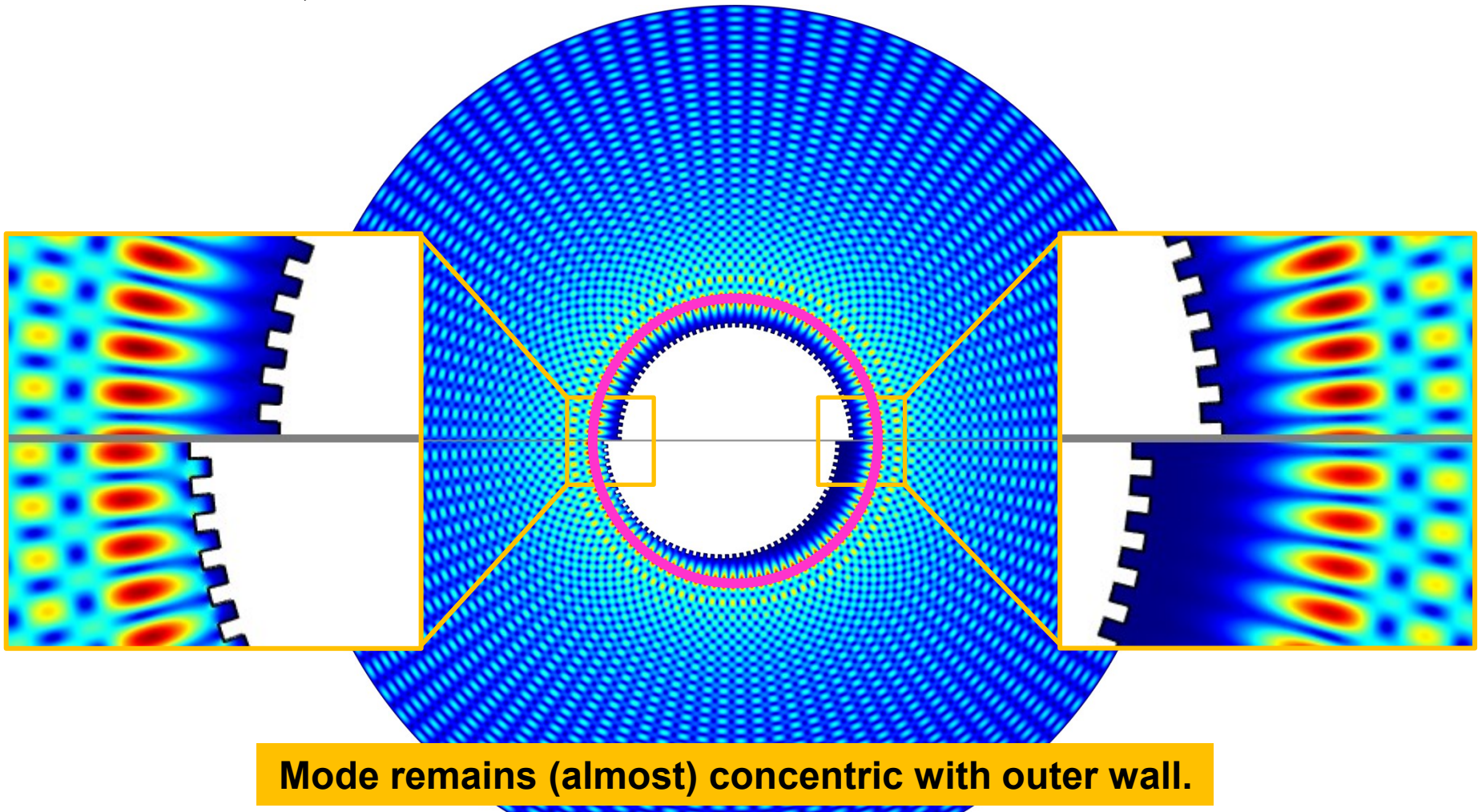
# Effects of Misalignment 2: Voltage Depression

- Mode  $TE_{49,29}$  at 0.0 mm / 1.0 mm misalignment



# Effects of Misalignment 2: Voltage Depression

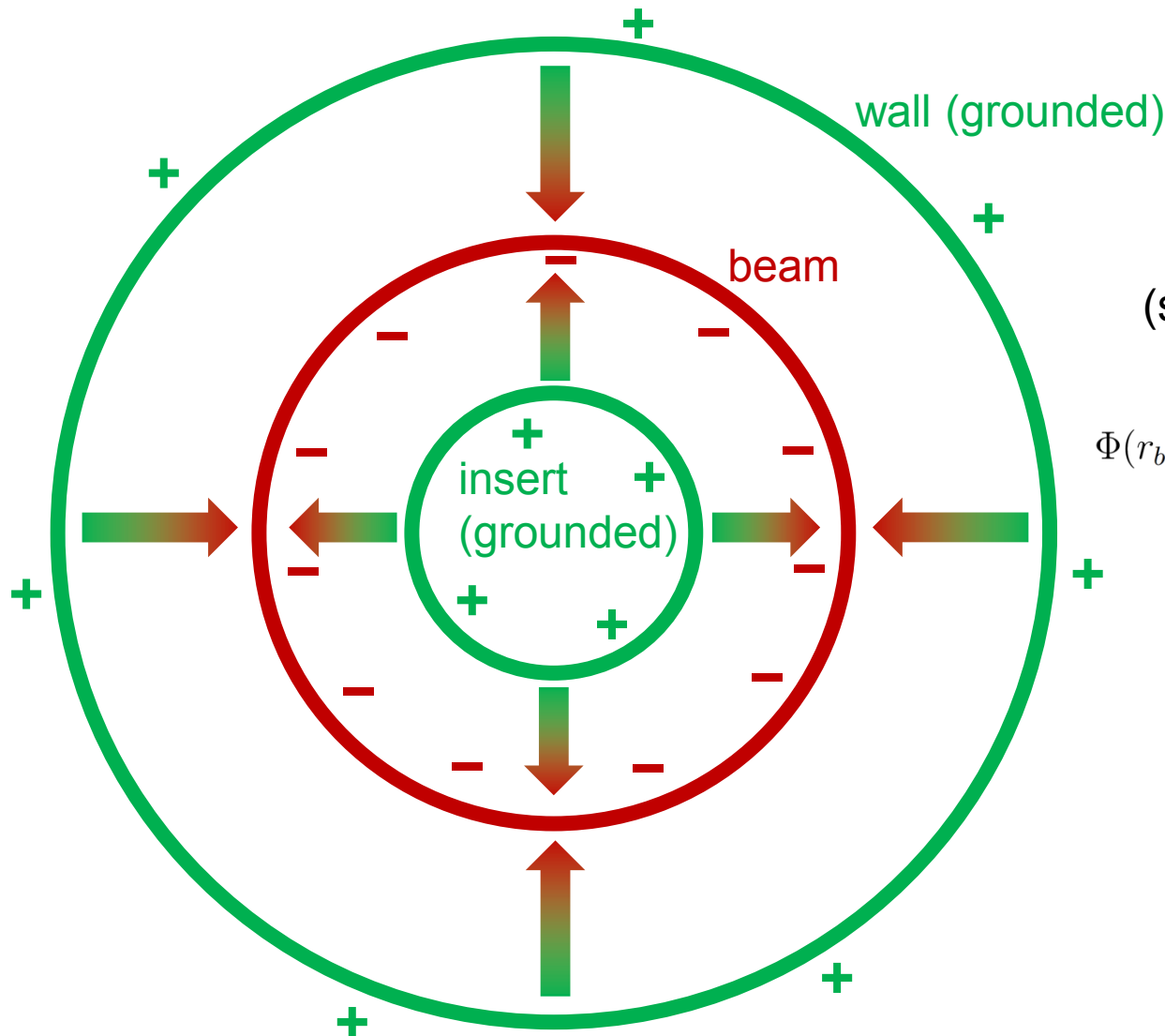
- Mode  $TE_{49,29}$  at 0.0 mm / 1.0 mm misalignment



**Mode remains (almost) concentric with outer wall.**

**For optimum coupling, the beam has to be concentric to the modes.**

# Effects of Misalignment 2: Voltage Depression

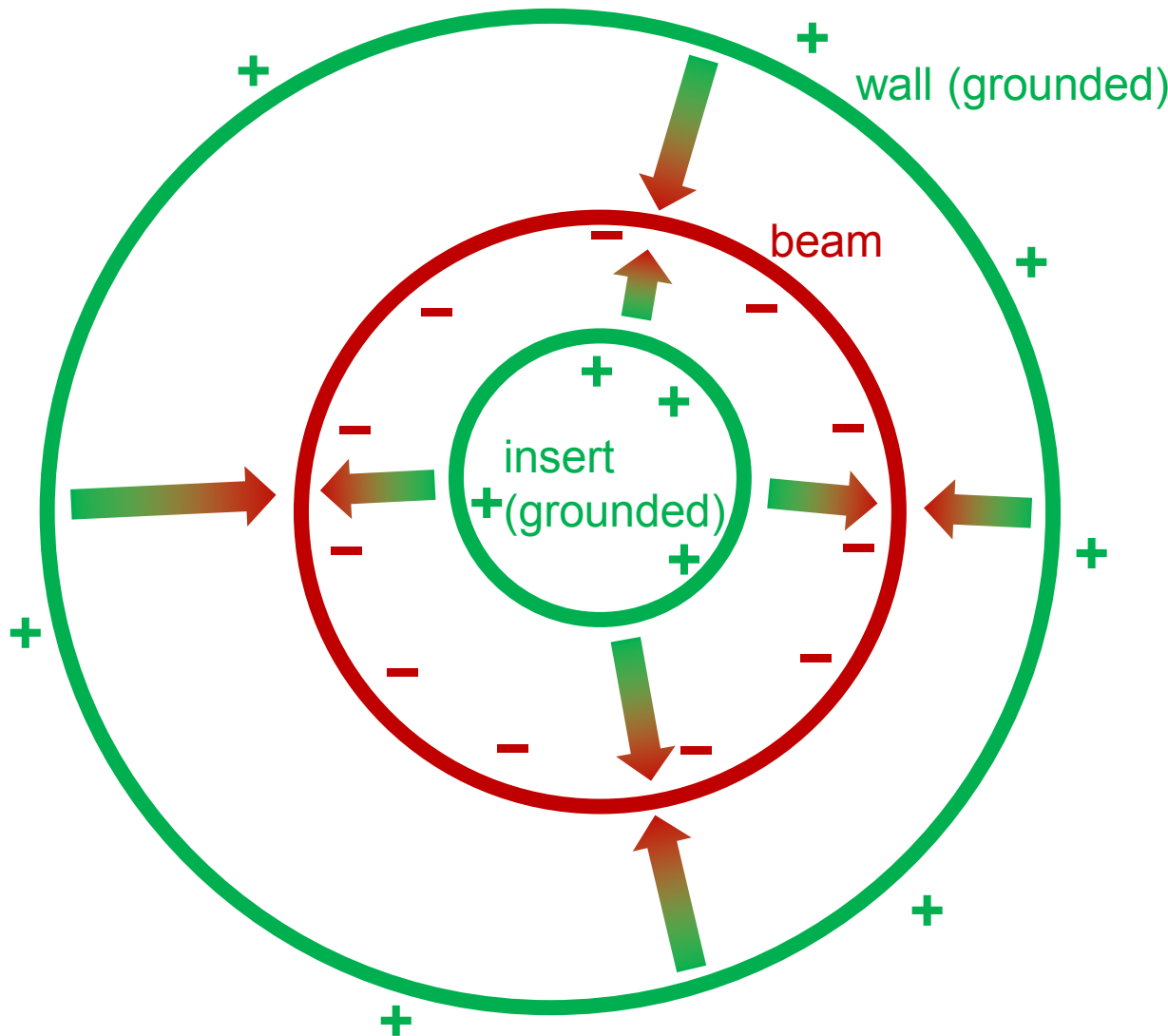


Gauss' Law  
(symmetric problem) →

$$\Phi(r_b) = \frac{\kappa_b}{2\pi\epsilon_0} \frac{\ln(r_b/R_O) \ln(r_b/R_I)}{\ln(R_O/R_I)}$$

$R_I$ : insert radius  
 $r_b$ : beam radius  
 $R_O$ : wall radius  
 $\kappa_b$ : beam line charge

# Effects of Misalignment 2: Voltage Depression



asymmetric problem

?

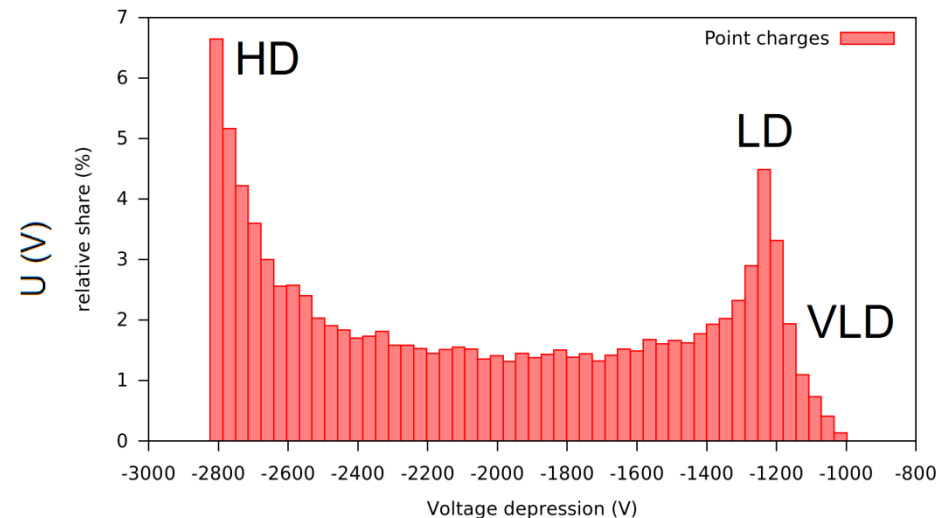
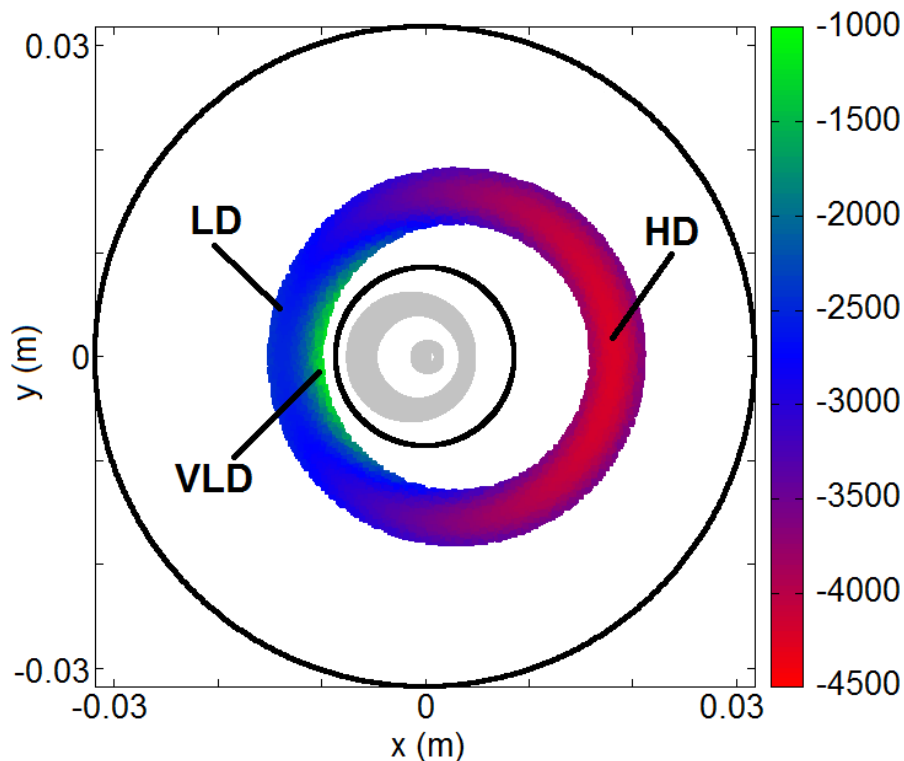
*depends on azimuthal coordinate*

- $R_i$ : insert radius
- $r_b$ : beam radius
- $R_o$ : wall radius
- $\kappa_b$ : beam line charge
- $D_{ib}$ : insert-beam misal.
- $D_{io}$ : insert-wall misal.

# Description of Misalignment: Voltage Depression

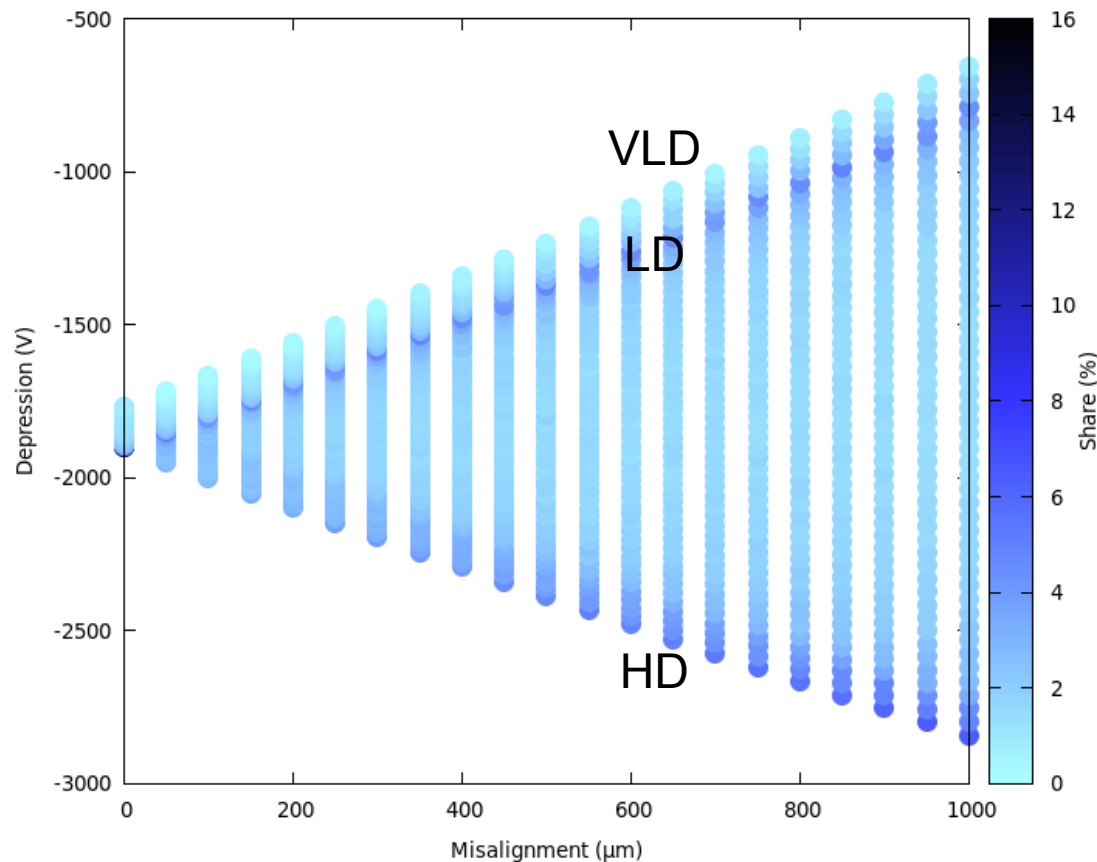
... see <http://dx.doi.org/10.1109/TED.2016.2592533> ...

■ Test design (very thick, misaligned beam):



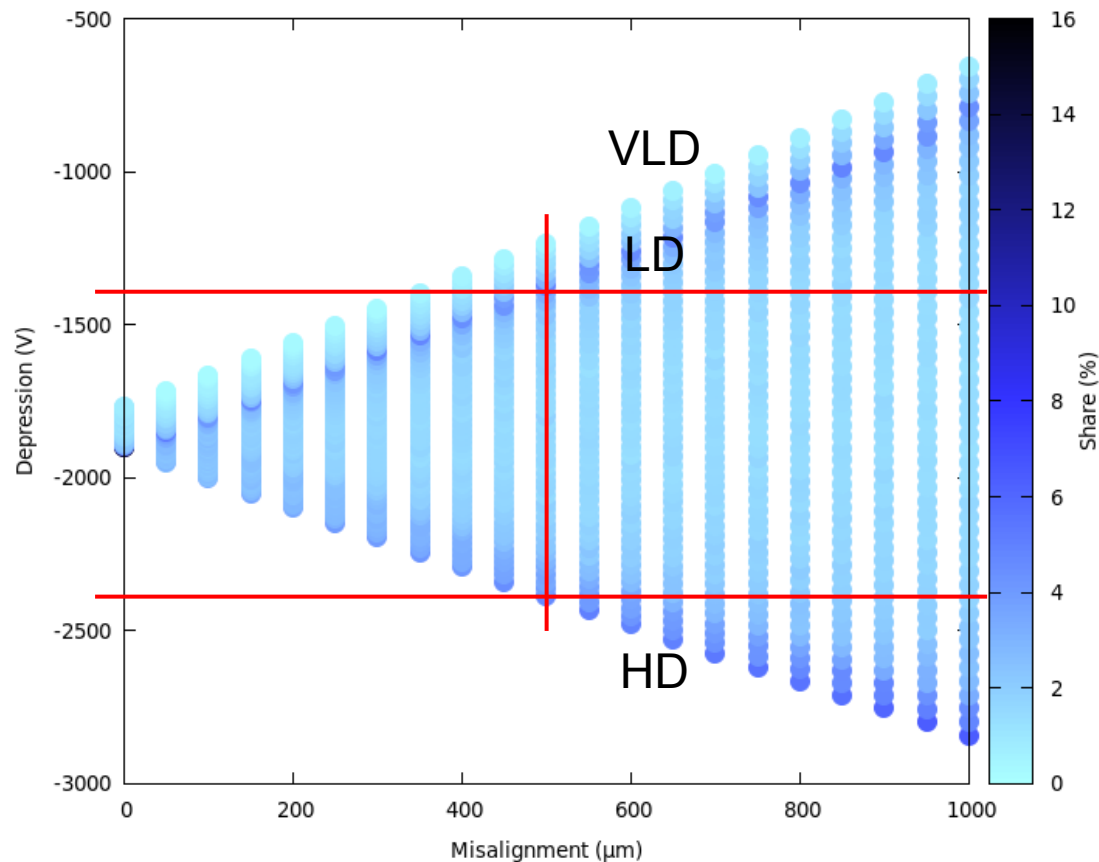
# Description of Misalignment: Voltage Depression

- TE<sub>49,29</sub> mode gyrotron design (misaligned insert+beam):



# Description of Misalignment: Voltage Depression

- TE<sub>49,29</sub> mode gyrotron design (misaligned insert+beam):



1 keV (1.17 %)  
energy variation

# Conclusion and Outlook

Two new codes for interaction simulation input:

- **SCNCHIMP**: mode content of cavity with misaligned insert
  - Interface to interaction code **Euridice** (K.A. Avramidis) is written
  - Additional features: tilted insert, thick electron beam, B-field profile
  - Finished; testing is ongoing
- **WickedQueen**: electron energy spread in cavity with misalignment
  - Article on principle and benchmarking published (IEEE Trans. Electron Devices **63**(9), 3740-3746, Sept. 2016)

Next steps:

- **Euridice** interaction simulations with
    - Energy spread input from **WickedQueen**
    - Mode input from **SCNCHIMP**
- *Determination of maximum allowed misalignment.*

# Questions/Discussion

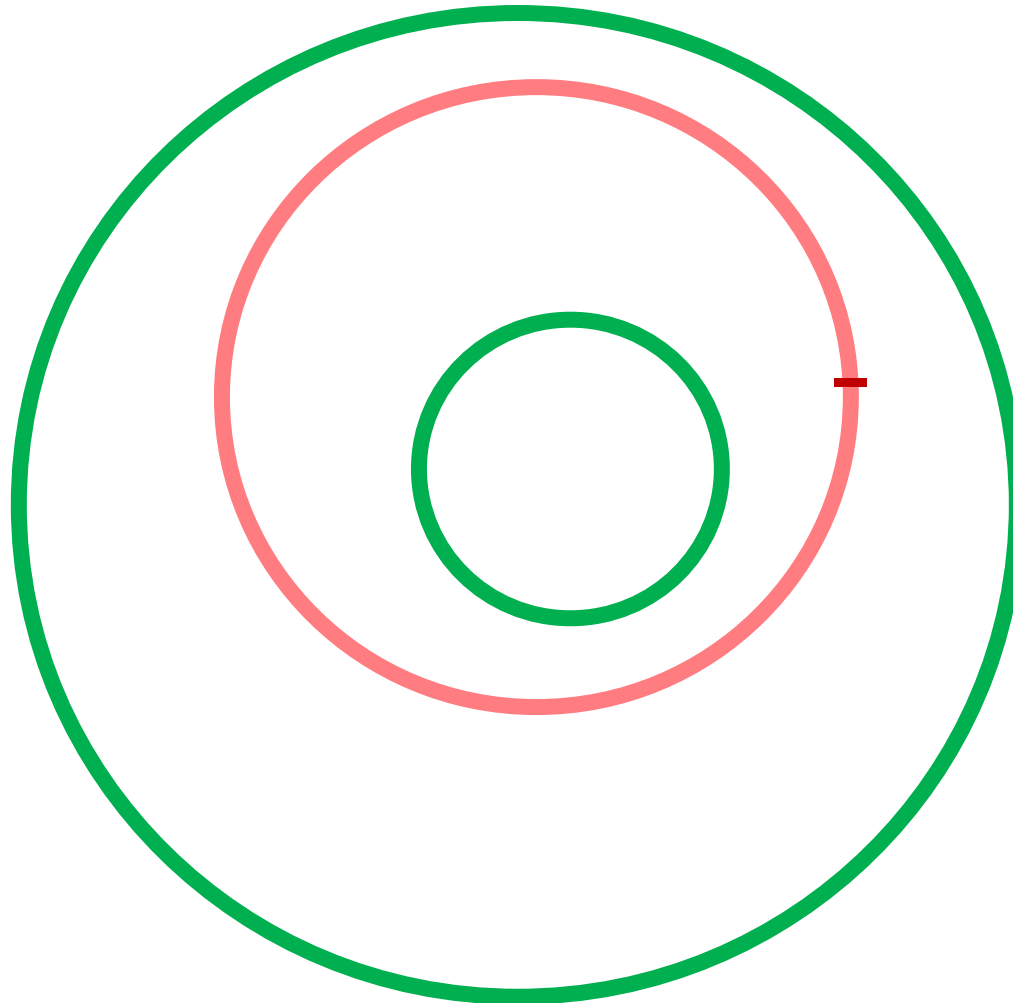


# BACKUP SLIDES

# Description of Misalignment: Voltage Depression

- Basic idea:

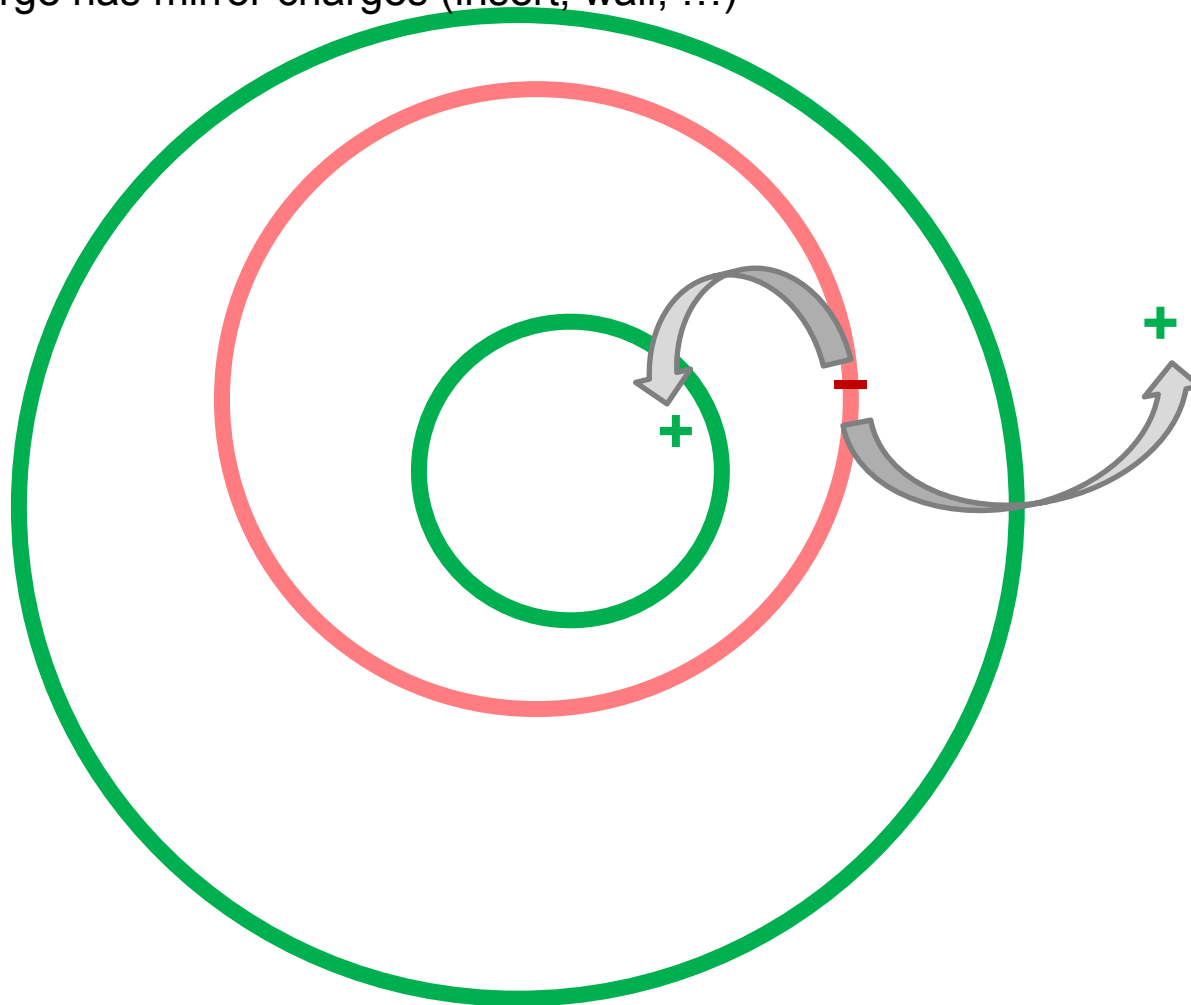
- Describe electron beam as (large) number of point-charges



# Description of Misalignment: Voltage Depression

## ■ Basic idea:

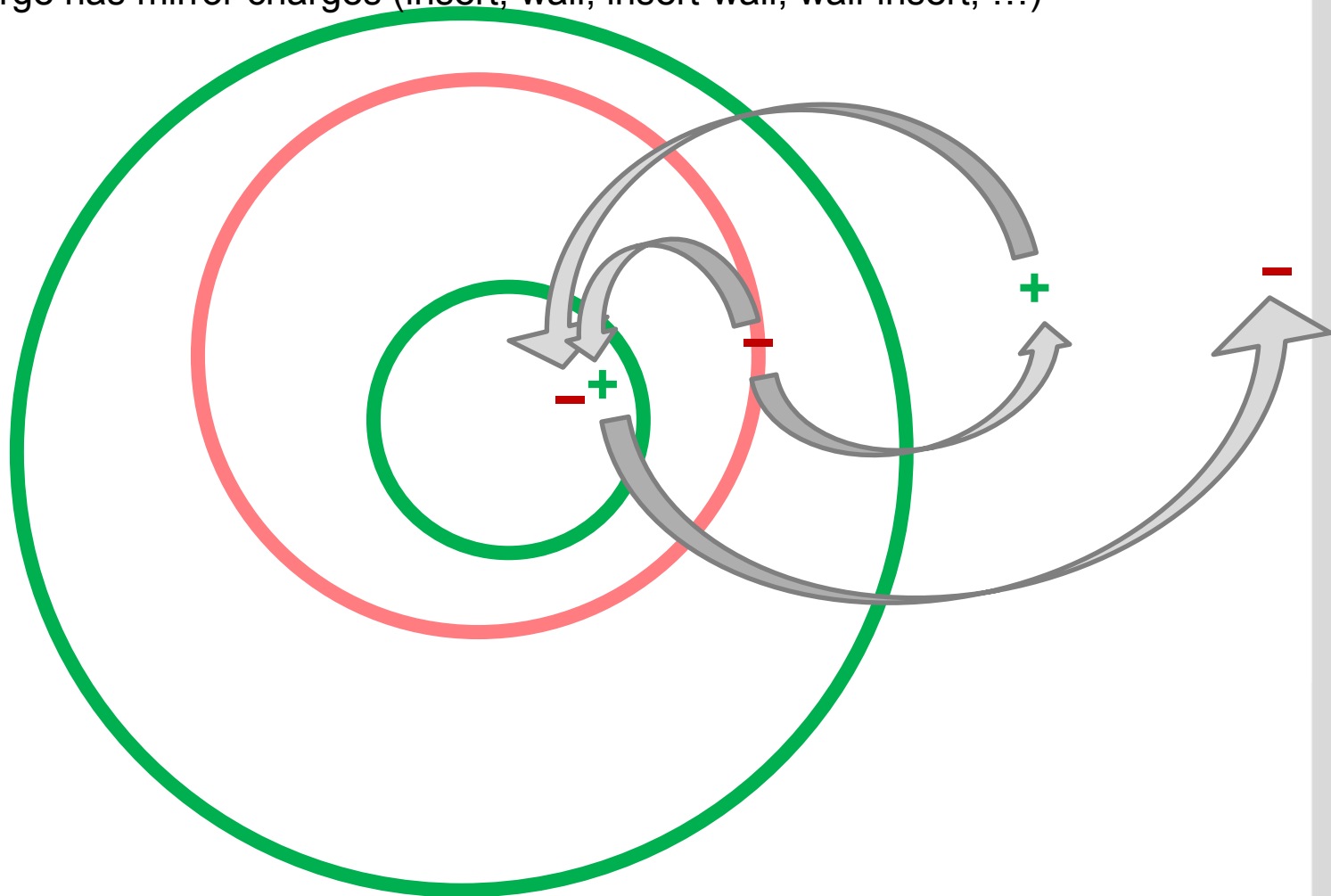
- Describe electron beam as (large) number of point-charges
- Each charge has mirror charges (insert; wall; ...)



# Description of Misalignment: Voltage Depression

## ■ Basic idea:

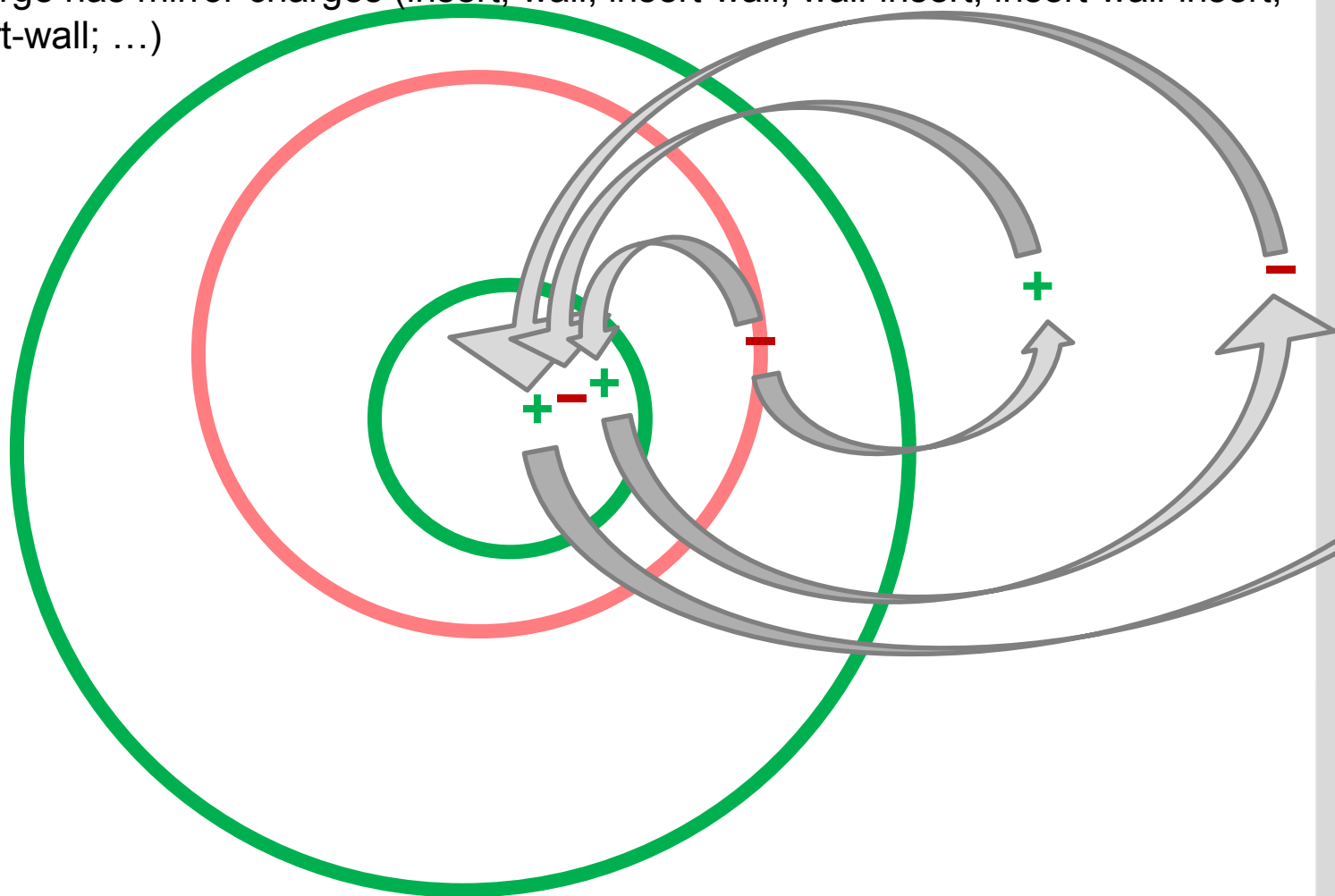
- Describe electron beam as (large) number of point-charges
- Each charge has mirror charges (insert; wall; insert-wall; wall-insert; ...)



# Description of Misalignment: Voltage Depression

## ■ Basic idea (seems to fail):

- Describe electron beam as (large) number of point-charges
- Each charge has mirror charges (insert; wall; insert-wall; wall-insert; insert-wall-insert; wall-insert-wall; ...)



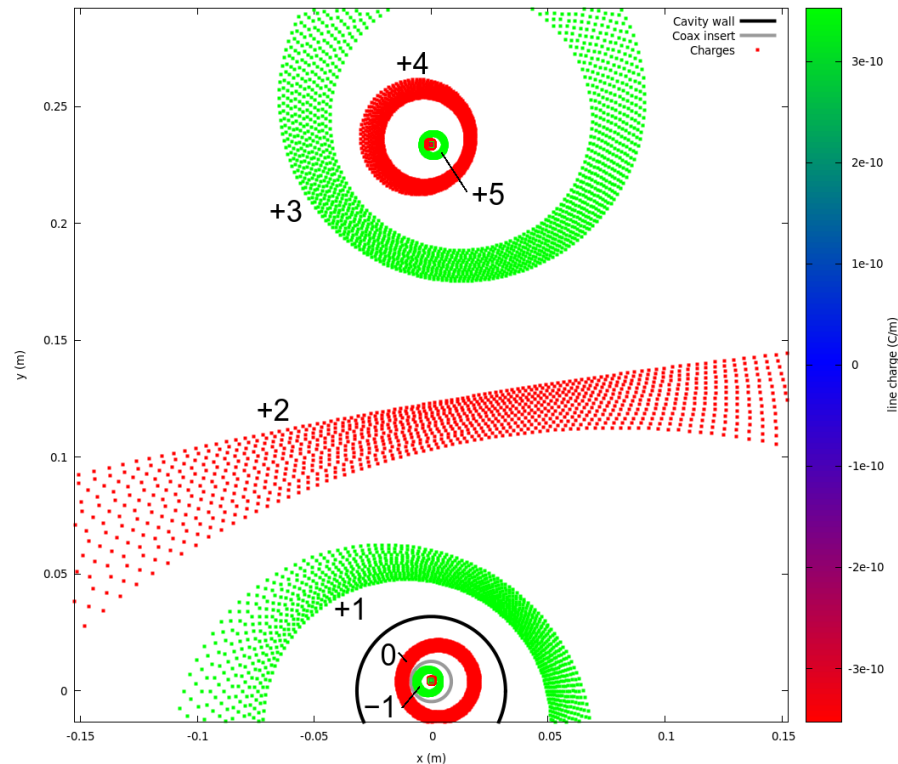
# Description of Misalignment: Voltage Depression

## ■ Basic idea (which does not fail):

- Describe electron beam as (large) number of point-charges
- Each charge has mirror charges (insert; wall; insert-wall; wall-insert; insert-wall-insert; wall-insert-wall; ...) → infinite number

## ■ But:

- Mirror charges converge to two distinct points of the geometry  
→ series can be truncated



# Description of Misalignment: Voltage Depression

- Basic idea (which does not fail):
  - Describe electron beam as (large) number of point-charges
  - Each charge has mirror charges (insert; wall; insert-wall; wall-insert; insert-wall-insert; wall-insert-wall; ...) → infinite number
- But:
  - Mirror charges converge to two distinct points of the geometry → series can be truncated
  - Axisymmetric beams need only a few (~20) mirror charges in total

