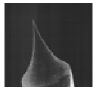
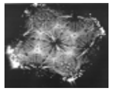
 High-Electric-Field Nanoscience 

# FIELD ELECTRON EMISSION THEORY FOR VACUUM ELECTRONICS


## Richard G. Forbes

Advanced Technology Institute &
Dept. of Electrical and Electronic Engineering,
University of Surrey



ITG Vacuum Electronics Workshop,
Bad Honnef, September 2018

# Primary aim of talk



Talk relates primarily to core theory of field electron emission (FE).
Its primary aim is to

- indicate some theoretical progress made in last ten years or so.

Talk provides "update" for people interested in electron sources, but who are not closely linked to modern developments in FE theory.

# Relevance of FE theory



***Why work on FE theory?***

- **FE is an enduring part of physics, and contributes to some enduring technology (e.g., electron microscopes), and to some enduring technological problems (e.g., vacuum breakdown). In particular:**
- **FE theory is needed in order to interpret experimental results and hence characterize emitters.**
- **Accurate FE theory is needed in order to carry out accurate simulations of various kinds.**

***Main field-electron-source technical contexts currently of interest are:***

- **Single tip sources (STFEs):** **Electron microscopes, etc.**
- **Large-area sources (LAFEs):** **X-ray generators, Microwave generators.**

***Field electron emission is also part of the story when electrical breakdown effects inhibit the successful development and operation of high-gradient particle accelerators, at organizations such as CERN.***

# *Structure of talk*



1. **Introductory issues**
2. **Transmission regimes and emission-current-density regimes**
3. **The classification of Fowler-Nordheim-type equations**
4. **Definition of area-like quantities**
5. **The theory of the principal SN barrier function v**
6. **Theory of non-ideal devices/systems**
7. **What next**

# *Energy terminology*



**Forwards =**

**Direction normal to and away from emitter surface**

**[also called "normal direction"]**

**[Distance in forwards direction is denoted by $z$ and measured from emitter's electrical surface]**

**Forwards energy $E_n$ =**

**Total electron energy associated with forwards direction**

**[relative to any arbitrary but specified energy reference zero]**

**$w$ = Forwards energy relative to top of barrier**

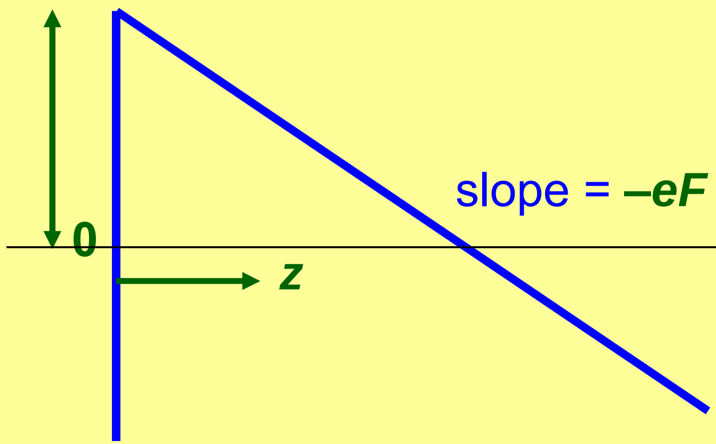
**$W$ = Forwards energy relative to base of conduction band.**

# *Two special barrier forms*



**Two well-known special barrier forms exist:**

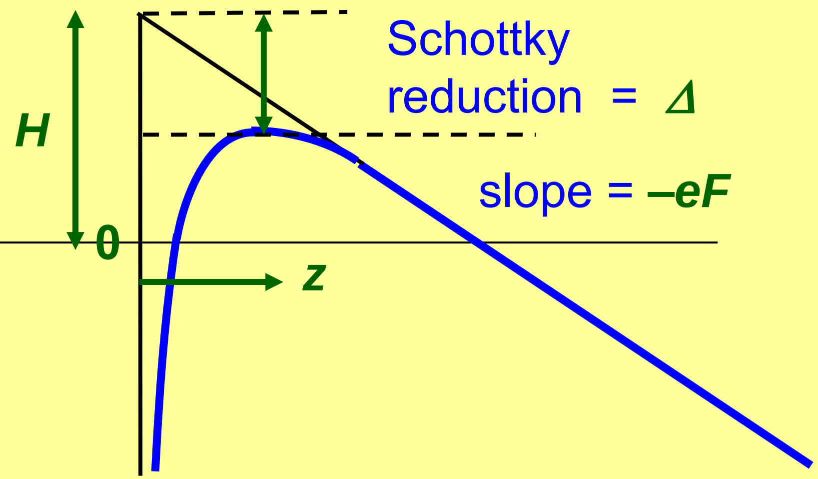
## Exactly triangular (ET) barrier




$$M(z) = H - eFz$$

used by **Fowler & Nordheim (FN)** in 1928

## Schottky-Nordheim (SN) barrier




$$M(z) = H - eFz - e^2/16\pi\varepsilon_0 z$$

used by **Murphy & Good (MG)** in 1956

# Scaled barrier field *f*



When the Schottky reduction $\Delta$ is equal to the work function $\phi$, we have

$$\Delta = \phi = c_S F_R^{1/2} = (e^3/4\pi\varepsilon_0)^{1/2} F_R^{1/2},$$

where $F_R$ [$= c_S^{-2}\phi^2$] is the reference field needed to reduce to zero a barrier of zero-field height $\phi$. [For $\phi = 4.50$ eV, we get $F_R \approx 14.1$ V/nm.]

The scaled barrier field $f$ is defined by

$$f = F / F_R = c_S^2 \phi^{-2} F .$$

This dimensionless parameter $f$ plays an important role in modern FE theory.

# Transmission regimes
# and
# Emission current density regimes

# *Transmission terminology*



**In relation to transmission, my nomenclature now is:**

**Transmission =**
**Escape of an entity *across* a potential-energy barrier.**

**Tunnelling =**
**Wave-mechanical escape *below* the top of the barrier.**

**Flyover =**
**Wave-mechanical escape *above* the top of the barrier.**

**Classical transmission =**
**Escape *greatly above* the top of the barrier, at a level where surface reflection effects are negligibly small, and transmission probability $D \approx 1$.**
**[In practice, typically 5 eV or more above the barrier top.]**

# *Regime terminology*



**A transmission regime =**

**Region of parameter-space (typically field and forwards energy) where particular effects determine transmission, or a particular formula for transmission probability *D* is an adequate approximation.**

**An emission-current-density (ECD) [or "emission"] regime =**

**Region of parameter-space (typically field and temperature, for given work-function) where a particular formula for local ECD *J* is an adequate approximation.**

# New calculations for ET barrier transmission regimes

PROCEEDINGS OF THE ROYAL SOCIETY A | MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

## Transmission coefficients for the exact triangular barrier: an exact general analytical theory that can replace Fowler & Nordheim's 1928 theory

Richard G. Forbes and Jonathan H. B. Deane

*Proc. R. Soc. A* 2011 **467**, 2927-2947 first published online 18 May 2011
doi: 10.1098/rspa.2011.0025

# *ET barrier – exact formula for $D^{\mathrm{ET}}$*

**For the exactly triangular barrier, wave-matching at the emitter surface leads to the exact general formula (for all fields and all forwards energies)**

$$D^{\mathrm{ET}} = \frac{1}{\frac{1}{2} + \frac{1}{4}\pi\omega(A^2 + B^2) + \frac{1}{4}\pi\omega^{-1}(A'^2 + B'^2)}$$

**where *A*, *B* are the values of the Airy functions Ai, Bi, and *A′*, *B′* are the values of the derivatives of Ai, Bi, all evaluated at the emitter surface.**

**The dimensionless parameter $\omega$ is given by an expression of the form (where $c_\kappa$ is an universal constant):**

$$\omega = c_\kappa W^{1/2}/F^{1/3} = [1.723903 \text{ eV}^{-1/2} (\text{V/nm})^{1/3}] \ (W^{1/2}/F^{1/3}) .$$

**where *W* is forwards energy measured relative to the base of the conduction band.**

# ET barrier – approximate formulae for $D^{ET}$



**The transmission regimes and good working formulae (for $D^{ET}$) are:**

**1) For deep tunnelling (DT) [$w$<<0]**

**the original Fowler-Nordheim approximate formula:**

$$D^{\mathrm{ET}} \approx P^{\mathrm{FN}} \exp[-bH^{3/2}/F] \;=\; \{4W^{1/2}H^{1/2}/(W+H)\}\; \exp[-bH^{3/2}/F]$$

**2) For the barrier top (BT) regime [$w$ ~ 0]:**

$$D^{\mathrm{ET}} \approx \frac{1}{\frac{1}{2} + c_0 F^{-1/3} W^{1/2} + c_\infty F^{1/3} W^{-1/2} - c_1 F^{-1} \chi W^{-1/2} w}$$

**where $c_0$, $c_\infty$, $c_1$ are constants with known values.**

**3) For high flyover (HF) [$w$>>0]:**

$$D^{\mathrm{ET}} \approx \frac{4W^{1/2}w^{1/2}}{(W^{1/2}+w^{1/2})^2}$$

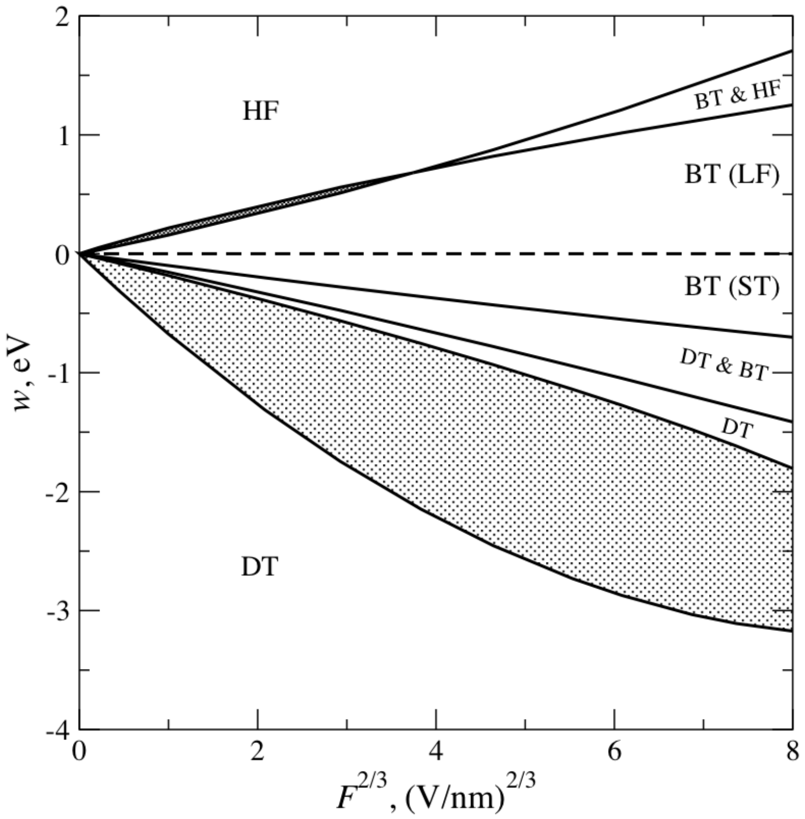
**[This is also the formula for transmission across a rectangular step.]**

# *ET-barrier transmission regimes*



**CT regime**

**above *w*= 5 eV**




**Classical transmission regime (CT)**
**mostly classical transmission ($D \approx 1$)**

**Surface-reflection regime (SR)**
**mostly high flyover (HF)**

**Barrier-top regime (BT)**
**low flyover (LF) + shallow tunnelling (ST)**

**Deep tunnelling (DT)**
**all deep tunnelling**

**Boundaries represent 10% errors.**

# SN-barrier: Transmission & ECD regimes



**Transmission regimes**

**Classical transmission regime**

**Surface-reflection regime**

**Barrier-top regime**

**Deep tunnelling**

# *SN-barrier: Transmission & ECD regimes*



| Transmission regimes | ECD regimes (Swanson/Bell/Forbes) |
|---|---|
| Classical transmission regime | *High-$T$ (low-$F$) limit =* Classical thermal electron emission (CTE) |
| Surface-reflection regime | Quantum-mechanical thermal electron emission (QMTE) |
| Barrier-top regime | Barrier-top electron emission (BTE) [or "extended Schottky"] |
| Deep tunnelling | Fowler-Nordheim FE (FNFE) [or "cold FE" (CFE)] |
| | Low-$T$ limit = Zero-$T$ FNFE |

# SN-barrier ECD regimes



| Old View | ECD regimes (Swanson/Bell/Forbes) |
|---|---|
| | *High-T (low-F) limit =* Classical thermal electron emission (CTE) |
| Over the barrier (Classical) "Thermionic" | Quantum-mechanical thermal electron emission (QMTE) |
| | Barrier-top electron emission (BTE) [or "extended Schottky"] |
| Through the barrier (Tunnelling) "Field emission" | Fowler-Nordheim FE (FNFE) [or "cold FE" (CFE)] |
| | Low-*T* limit = Zero-*T* FNFE |

# *SN-barrier ECD regimes, & practical devices*



| Old View | ECD regimes (Swanson/Bell/Forbes) | Commercial items |
|---|---|---|
| | *High-T (low-F) limit =* Classical thermal electron emission (CTE) | No devices |
| Over the barrier (Classical) "Thermionic" | Quantum-mechanical thermal electron emission (QMTE) | Thermionic emitter |
| | Barrier-top electron emission (BTE) [or "extended Schottky"] | Schottky emitter |
| Through the barrier (Tunnelling) "Field emission" | Fowler-Nordheim FE (FNFE) [or "cold FE" (CFE)] | Field emitter |
| | Low-*T* limit = Zero-*T* FNFE | |

# Current-density formulae for ECD regimes



| Commercial items | Operating equation |
| --- | --- |
| | For clarity below, define $J_{kRS} \equiv \exp[(e^3F/4\pi\varepsilon_0)^{1/2}/k_BT] \exp[-\phi/k_BT]$ |
| No devices | **Classical Richardson-Schottky (RS) equation** $J = A_{0R} J_{kRS}$ |
| Thermionic emitter | **Quantum-mechanical RS equation** $J = D_{eff} A_{0R} J_{kRS}$ |
| Schottky emitter | **Barrier-top electron emission equation** $J = \{(\pi q)/\sin(\pi q)\} J_{kRS}$ |
| Field emitter | **Fowler-Nordheim-type equation, e.g.** $J = \lambda^{SN} a\phi^{-1}F^2 \exp[-v_F b\phi^{3/2}/F]$ |

# The classification
# of
# Fowler-Nordheim-type equations

# *Fowler-Nordheim-type equations*



**A Fowler-Nordheim-type (FN-type) equation is any FNFE equation with the mathematical form**

$$Y = C_{YX} X^2 \exp[-B_X / X] ,$$

**where:** $X$ **is any FE independent variable (e.g., a field or a voltage);**

$Y$ **is any FE dependent variable (e.g., a current or current density);**

$B_X$ **is a function related to choice of** $X$ **and barrier form;**

$C_{YX}$ **is a function related to other choices, including** $X$, $Y$, **and** $B_X$ **.**

$B_X$ **and** $C_{YX}$ **are NOT constants (except in the most elementary models).**

**The core theoretical forms of FN-type equation (those derived directly from theory) give local emission current density (ECD)** $J$ **in terms of local work function** $\phi$ **and (the magnitude** $F$ **of) local barrier electrostatic field.**

# *The elementary FN-type equation*



**The simplest core FN-type equation is the elementary FN-type equation:**

$$J^{\mathrm{el}} \;=\; a\phi^{-1}F^{2}\exp[-b\phi^{3/2}/F] \; .$$

**where *a* and *b* are the FN constants.**

**This is based on tunnelling through an exactly triangular (ET) barrier, and is a simplification of the original 1928 FN-type equation.**

**This equation above is too simple to describe real situations. Hence, it has to be generalised, in TWO ways.**

# *The concept of "kernel current density"*

(1) The elementary equation

- neglects exchange-and-correlation (XC) effects (usually modelled as image effects);
- is not adequately valid for highly curved emitters.

We formally include both effects with a barrier form correction factor, here for a general barrier (GB).

A general barrier of zero-field height $\phi$ has a correction factor $\nu_F^{GB}$ ($\nu$ = "nu"); the resulting equation is

$$J_k^{GB} = a\phi^{-1}F^2 \exp[-\nu_F^{GB} b\phi^{3/2}/F].$$

$J_k^{GB}$ is a mathematical quantity that can be calculated *exactly* for a given barrier form, when $\phi$ and $F$ are given.

I call $J_k^{GB}$ the kernel current density for the chosen barrier form "GB".

# *Local pre-exponential correction factor*



(2) To allow for other corrections, it is necessary to include a local pre-exponential correction factor $\lambda^{GB}$.

Thus the physical local ECD $J^{GB}$ is given by

$$J^{GB} = \lambda^{GB} J_k^{GB} = \lambda^{GB} (a\phi^{-1}F^2) \exp[-\nu_F^{GB} b\phi^{3/2}/F].$$

The factor $\lambda^{GB}$ allows *formally* for corrections due to all of:

- improved tunnelling theory that includes a tunnelling pre-factor;
- more accurate integration over emitter electron states;
- temperature effects;
- effects due to the use of atomic-level wave-functions;
- effects related to non-free-electron band-structure;
- any other operating physical effect not specifically considered;
- any unrecognized theoretical inadequacy.

The equation above is the core general-barrier FN-type equation.

# *Local pre-exponential correction factor*



(2) To allow for other corrections, it is necessary to include a local pre-exponential correction factor $\lambda^{\mathrm{GB}}$.

Thus the physical local ECD $J^{\mathrm{GB}}$ is given by

$$J^{\mathrm{GB}} \quad = \quad \lambda^{\mathrm{GB}}\, J_{\mathrm{k}}^{\mathrm{GB}} \quad = \quad \lambda^{\mathrm{GB}}\,(a\phi^{-1}F^2)\,\exp[-\nu_{\mathrm{F}}^{\mathrm{GB}} b\phi^{3/2}/F].$$

The factor $\lambda^{\mathrm{GB}}$ allows *formally* for corrections due to all of:

- improved tunnelling theory that includes a tunnelling pre-factor;
- more accurate integration over emitter electron states;
- temperature effects;
- effects due to the use of atomic-level wave-functions;
- effects related to non-free-electron band-structure;
- any other operating physical effect not specifically considered;
- any unrecognized theoretical inadequacy.

The equation above is the core general-barrier FN-type equation.

Values for $\lambda$ are not reliably known for any physically realistic barrier.

# Equation complexity levels



Historically, many different assumptions/models have been used to obtain expressions for $\nu_F^{GB}$ and $\lambda^{GB}$.

The complexity level of a FN-type equation is decided by the choices of:

(a) barrier form (which determines $\nu_F$); and

(b) what effects/approximations to include in $\lambda$.

For planar emitters, the main complexity levels used historically and currently are shown in the following table.

# Most significant planar complexity levels



TABLE 1. Complexity levels of core planar Fowler-Nordheim-type equations.

| Name | Date | $\lambda^{GB} \rightarrow$ | Barrier form | $\nu_F^{GB} \rightarrow$ | Note |
|---|---|---|---|---|---|
| Elementary | ? | 1 | ET | 1 | a |
| Original | 1928 | $P_F^{FN}$ | ET | 1 | b |
| Fowler-1936 | 1936 | 4 | ET | 1 | |
| Extended elementary | 2015 | $\lambda^{ET}$ | ET | 1 | |
| Dyke-Dolan | 1956 | 1 | SN | $v_F$ | c |
| Murphy-Good (zero temperature) | 1956 | $t_F^{-2}$ | SN | $v_F$ | c |
| Murphy-Good (finite temperature) | 1956 | $\lambda_T t_F^{-2}$ | SN | $v_F$ | d |
| Orthodox | 2013 | $\lambda^{SN0}$ | SN | $v_F$ | e |
| New-standard | 2015 | $\lambda^{SN}$ | SN | $v_F$ | |
| "Barrier-effects-only" | 2013 | $\lambda^{GB0}$ | GB | $\nu_F^{GB}$ | e |
| General | 1999 | $\lambda^{GB}$ | GB | $\nu_F^{GB}$ | |

[a] Earlier imprecise versions exist, but the first clear statement seems to be in 1999.

[b] $P_F^{FN}$ is the Fowler-Nordheim tunnelling pre-factor.

[c] $v_F$ and $t_F$ are appropriate particular values of the SN barrier functions v and t.

[d] $\lambda_T$ is the Murphy-Good temperature correction factor

[d] The superscript " $^0$ " indicates that the factor is to be treated mathematically as constant.

For details, see: R.G. Forbes et al., "Fowler-Nordheim plot analysis: a progress report", Jordan J. Phys. 8 (2015) 125; arXiv:1504.06134v7 .

Historically, around 15-20 different mathematical approximations have been used for the particular value $v_F$ of the principal SN barrier function v.

# Definition of area-like quantities

# Emission area definitions

To derive expressions for emission current, one needs to identify a characteristic point "C" on the emitter surface. In modelling, "C" is usually taken at the emitter apex. Parameters relating to "C" are subscripted "$_{\mathrm{C}}$".

An expression for total emission current $i_e$ is obtained by integrating over the emitter surface and writing result in form

$$i_e = \int J \, dA \equiv A_n J_C = A_n \lambda_C J_{kC} \,,$$

where $A_n$ is the notional emission area.

For all emitters, the value of $\lambda_C$ is uncertain, and for large-area field electron emitters (LAFEs) the value of $A_n$ is also uncertain. Having two parameters of uncertain value in an equation is unhelpful, so define a new parameter, the formal emission area $A_f$ by

$$A_f \equiv A_n \lambda_C \,.$$

# *Different roles of formal and notional parameters*



**We need both formal and notional area-like parameters, because:**

- **in appropriate circumstances ("where the emitting device/system is ideal and emission is orthodox"), good values of the formal parameters can deduced from experiment, using so-called Fowler-Nordheim plots;**
- **but the notional parameters appear in some existing theory.**

**In principle, the notional parameters are probably closer to "geometrical" area estimates, but (due to uncertainty in $\lambda_C$), accurate values of notional parameters cannot be deduced from experiment.**

**Values of formal emission area deduced from FN plots may sometimes look implausibly low.**

## Consequences of the $\lambda$-value problem



For the SN barrier, the value of the $\lambda^{SN}$ is thought (in 2018) to lie in the range

$$0.005 < \lambda^{SN} < 14 \text{ ,}$$

but this could be either an underestimate or an overestimate of the range of uncertainty.

In summary, two consequences of our lack of good knowledge of $\lambda$ are

- we cannot carry out accurate simulations of FE current densities and currents;
- we cannot accurately deduce values of emission area from FE experiments.

# [New understanding of]

# the theory of the

# principal SN-barrier function v

# *The special mathematical function* v($x$)



**Mathematically, the principal SN barrier function v is a *special mathematical function* that is a particular solution of a special equation identified by Deane and Forbes, namely**

$$x(1-x)\mathrm{d}^2W/\mathrm{d}x^2 = (3/16)W \,.$$

**This equation is itself a special case of the Gauss hypergeometric differential equation. Hence, I call $x$ the Gauss variable, and now write v($x$).**

# *The special mathematical function* v($x$)



Mathematically, the principal SN barrier function v is a *special mathematical function* that is a particular solution of a special equation identified by Deane and Forbes, namely

$$x(1-x)\mathrm{d}^2W/\mathrm{d}x^2 \ = \ (3/16)W \ .$$

This equation is itself a special case of the Gauss hypergeometric differential equation. Hence, I call $x$ the Gauss variable, and now write v($x$).

From detailed mathematical analysis, it can be shown that for a Schottky-Nordheim barrier of zero-field height $\phi$, the physical-modelling barrier form correction factor $\nu_F^{SN}$ is obtained from the special mathematical function v($x$) by setting $x$=$f$ (where $f$, as before, is scaled barrier field). That is:

$$\nu_F^{SN} \ = \ \mathrm{v}(f) \ .$$

# *The special mathematical function* v($x$)



Mathematically, the principal SN barrier function v is a *special mathematical function* that is a particular solution of a special equation identified by Deane and Forbes, namely

$$x(1-x)\mathrm{d}^2W/\mathrm{d}x^2 \;=\; (3/16)W \,.$$

This equation is itself a special case of the Gauss hypergeometric differential equation. Hence, I call $x$ the Gauss variable, and now write v($x$).

From detailed mathematical analysis, it can be shown that for a Schottky-Nordheim barrier of zero-field height $\phi$, the physical-modelling barrier form correction factor $\nu_F^{SN}$ is obtained from the special mathematical function v($x$) by setting $x$=$f$ (where $f$, as before, is scaled barrier field). That is:

$$\nu_F^{SN} \;=\; \mathrm{v}(f) \,.$$

This approach, which uses a mathematical variable $x$ and a modelling variable $f$, is intended to replace the older approach based on the Nordheim parameter $y$, which has to be interpreted as $\sqrt{x}$ or $\sqrt{f}$, as appropriate.

# *A simple good approximation for* v($x$) *or* v($f$)



The "Forbes-Deane" approximation for v($x$) is

$$v(x) \approx 1 - x + (1/6)x\ln x \,.$$

Over the range $0 \leq x \leq 1$, this expression is accurate to better than 0.33% .

Obviously, in terms of $f$ this becomes

$$v(f) \approx 1 - f + (1/6)f\ln f \,.$$

This approximation is more accurate than historical approximations of equivalent complexity ….

# Historical approximations for v(*f*) compared

**Table 1: APPROXIMATIONS COMPARED**

A1: ● $v_2 = 1 - f$ ,

A2: ● $v_A = 0.965 - 0.739f$,

A3: ● $v_{CM} = 0.956 - 1.062f$ ,

A4: ● $v_B = 1 - f[1+0.9\sin\{(1-f^{1/2})/2\}]$ ,

A5: ● $v_{SBHW} = 0.95 - f$

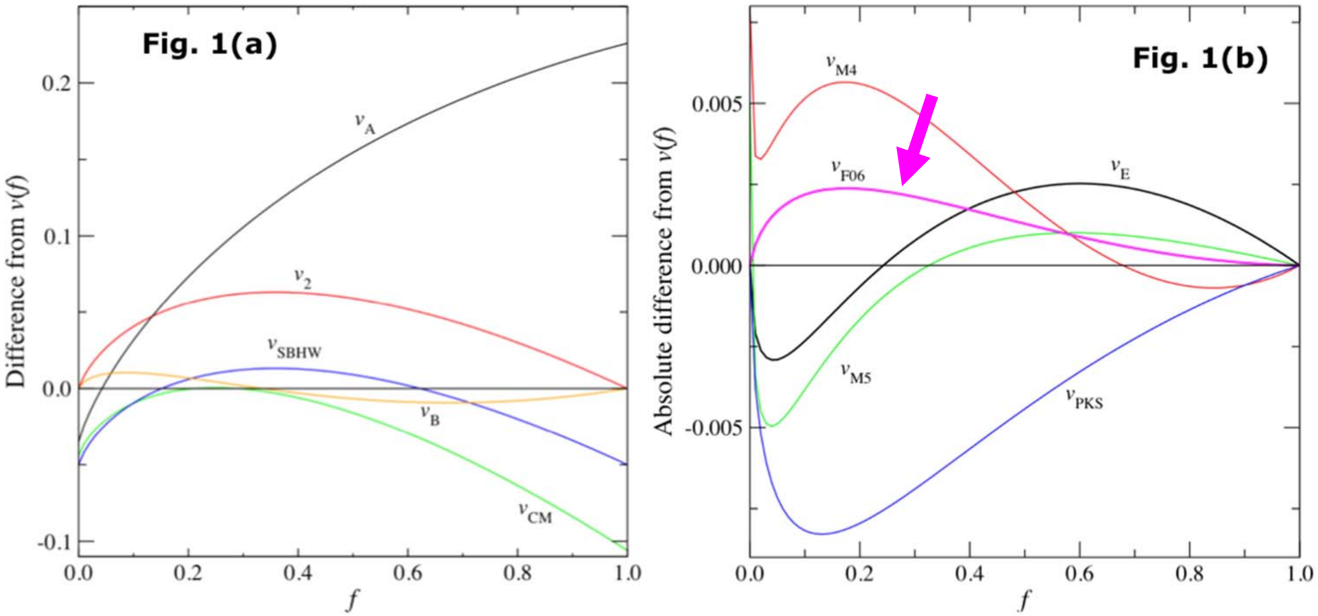
A6: ● $v_E = 1 - f^{0.845}$

A7: ● $v_{M4} = 1.008 - 0.118f^{1/2} - 1.14f + 0.25f^{3/2}$

A8: ● $v_{M5} = 1.0050 - 0.1654f^{1/2} - 1.0412f + 0.2320f^{3/2} - 0.0304f^2$

A9: ● $v_{PKS} = 1 - f^{0.82758}$

A10: ● $v_{F06} = 1 - f + (1/6) f \ln f$ [the new approximation].



# The exact series expansion for v($x$)



The lowest few terms of the exact series expansion for v($x$) are

$$\mathrm{v}(x) = 1 - \left(\frac{9}{8}\ln 2 + \frac{3}{16}\right)x - \left(\frac{27}{256}\ln 2 - \frac{51}{1024}\right)x^2 - \left(\frac{315}{8192}\ln 2 - \frac{177}{8192}\right)x^3 - \ldots$$
$$+ \left(\frac{3}{16} + \frac{9}{512}x + \frac{105}{16384}x^2 + \ldots\right) x \ln x$$

This series is derived from an exact mathematical statement of the analytical form of v($x$). This is too complicated to present here, but may be found in Deane and Forbes, *J. Phys. A: Math. Theor.* 41 (2008) 395301.

Almost certainly, the reason why the Forbes-Deane approximate formula works well is that it mimics the form of the lowest terms of the exact expansion.

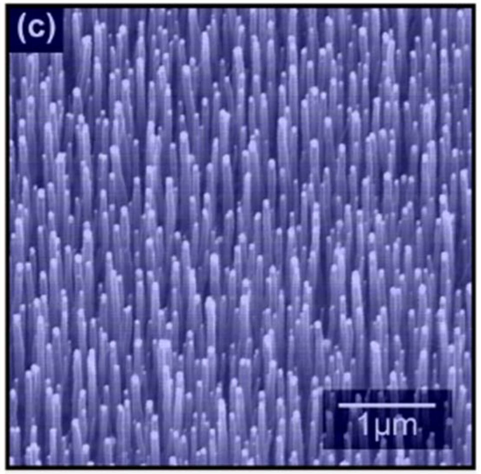
# Theory of non-ideal devices/systems

# Large-area field electron emitters



**A large-area field electron emitter (LAFE) has many/very-many individual emitter/emission site**

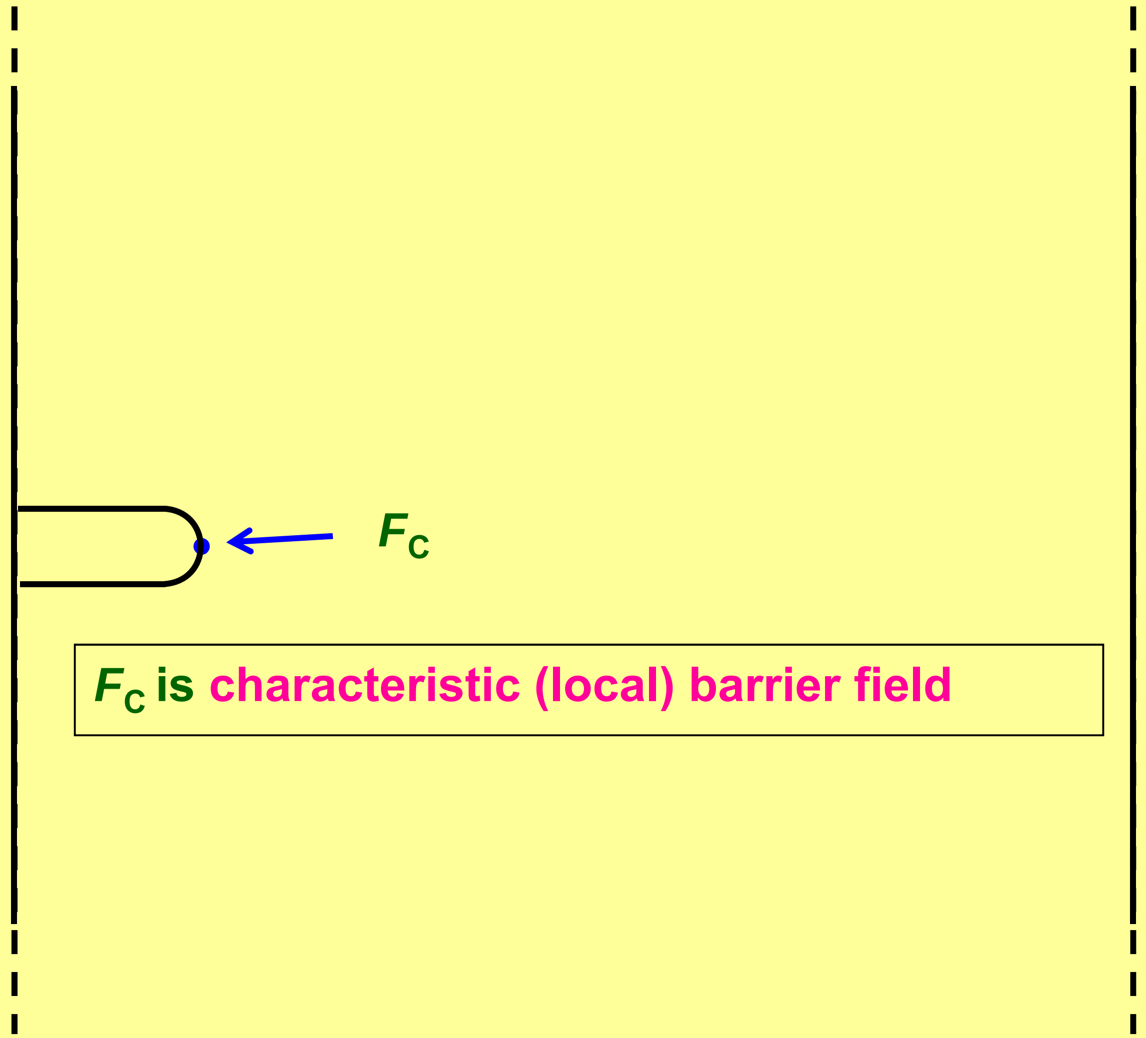
**[Carbon nanotube field emitter array shown]**
[Diagram: Courtesy: M.T. Cole et al.]




**With LAFEs, a commonly used characterization parameter in the characteristic macroscopic field enhancement factor (FEF) $\gamma_C$ [or $\beta$]. [Think of this as the FEF for a "typical long post".]**

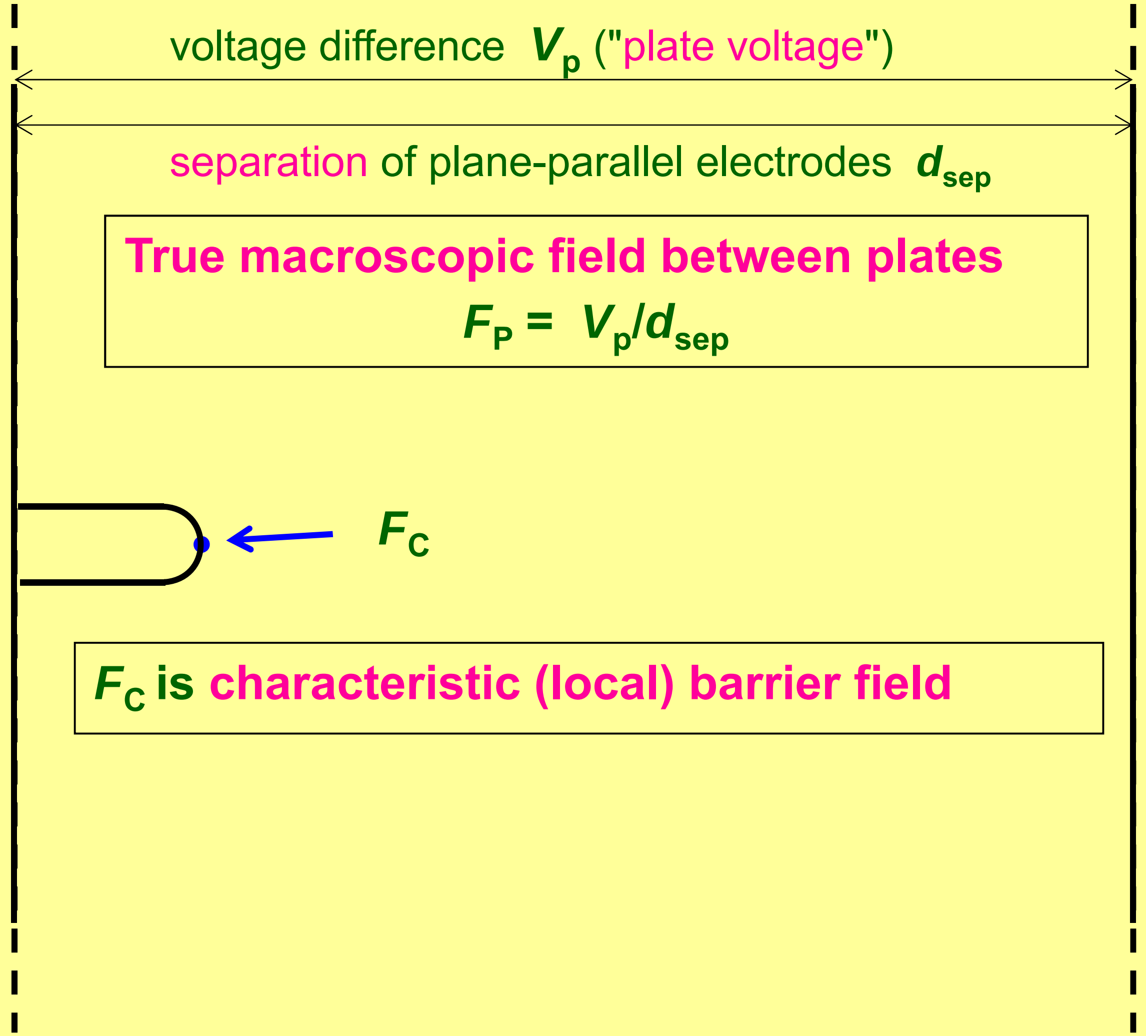
# Definition of "true FEF" for parallel plates






$F_C$ is characteristic (local) barrier field

# Definition of "true FEF" for parallel plates



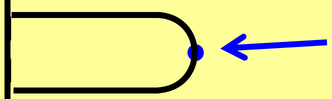
# Definition of "true FEF" for parallel plates

voltage difference $V_p$ ("plate voltage")

separation of plane-parallel electrodes $d_{sep}$

**True macroscopic field between plates**

$$F_P = V_p / d_{sep}$$



$F_C$

**$F_C$ is characteristic (local) barrier field**

**True characteristic macroscopic field enhancement factor (true-FEF) $\gamma_C$ is**

$$\gamma_C = F_C / F_P .$$

# The orthodoxy test



**For ideal FE devices/systems, experimental FEF estimates can be derived from Fowler-Nordheim plots (FN plots) by applying orthodox or elementary data-analysis theory.**

**For non-ideal FE devices/systems, use of ideal-device theory can lead to the extraction of spuriously large FEF-values.**

# *The orthodoxy test*



**For ideal FE devices/systems, experimental FEF estimates can be derived from Fowler-Nordheim plots (FN plots) by applying orthodox or elementary data-analysis theory.**

**For non-ideal FE devices/systems, use of ideal-device theory can lead to the extraction of spuriously large FEF-values.**

**An "orthodoxy test" has been devised, that can test whether a FN plot corresponds to a non-ideal device/system.**

**Tests on a sample of 17 published results, relating to emitters fabricated from various materials, found that 40% of published FEF values were spuriously large.**

# *The orthodoxy test*

**For ideal FE devices/systems, experimental FEF estimates can be derived from Fowler-Nordheim plots (FN plots) by applying orthodox or elementary data-analysis theory.**

**For non-ideal FE devices/systems, use of ideal-device theory can lead to the extraction of spuriously large FEF-values.**

**An "orthodoxy test" has been devised, that can test whether a FN plot corresponds to a non-ideal device/system.**

**Tests on a sample of 17 published results, relating to emitters fabricated from various materials, found that 40% of published FEF values were spuriously large.**

**A procedure called phenomenological adjustment has been introduced, to reduce these values down towards more realistic levels.**

# *Reasons for non-ideality*

There are various possible reasons for non-ideality of FE devices/systems, including:

- Series resistance in the measurement circuit.
- Field-dependent emitter geometry (due to Maxwell-stress effects).
- Current-dependent changes in work-function (due to adsorbate desorption as a result of heating).

Until very recently, it been thought that the most common cause would be series resistance.

# Reasons for non-ideality



There are various possible reasons for non-ideality of FE devices/systems, including:

- Series resistance in the measurement circuit.
- Field-dependent emitter geometry (due to Maxwell-stress effects).
- Current-dependent changes in work-function (due to adsorbate desorption as a result of heating).

Until very recently, it been thought that the most common cause would be series resistance.

Very recently, a further reason for non-ideality has been (re-)discovered, namely: current-dependence in the field enhancement factor.

It is now thought that this is likely to be a common cause of non-ideality.

# Notation for fields



In conventional classical electrostatics, the conventional symbol for classical electrostatic field is $E$ . When discussing electrostatic issues in field electron emission, the least confusing convention is to use classical electrostatic field.

However, for an operating field electron emitter the classical electrostatic field is negative. Hence, field electron emission theory uses a positive quantity equal to the negative or magnitude of classical electrostatic field, and refers to this quantity as the "field". This is sometimes known as the electron emission convention. FE theoreticians often prefer to use the symbol $F$ to denote this positive quantity.  The best approach defines $F$ by

$$F = -E .$$

However, many FE experimentalists use the symbol $E$ to denote this positive quantity.

Note that, in the following slides, $E$ denotes classical electrostatic field and is negative in sign.

# The role of electron thermodynamics

To understand the electrostatics of classical conductors, where the charge carrier is an electron, it is necessary to understand that the electrostatics of classical conductors is a branch of electron thermodynamics.

When no current is flowing, the condition for thermodynamic (and statistical mechanical) equilibrium is that the Fermi level be the same everywhere in the conductor.

In basic models, the simplifying assumption is usually made that the work function of all surfaces is the same.

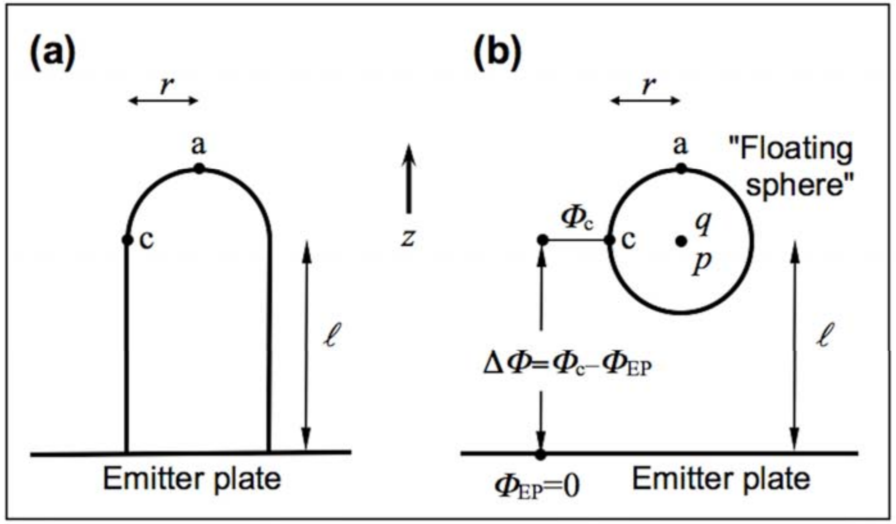
These two assumptions imply that the classical electrostatic potential $\Phi$ must be the same at all points "immediately outside" the surface of the classical conductor.

This is the underlying principle used in basic electrostatic discussions.

# *FSEPP model for field electron emitter*



**The floating-sphere-at-emitter-plate-potential (FSEPP) model is an approximate methodology for calculating the apex FEF $\gamma_a$ at point "a".**



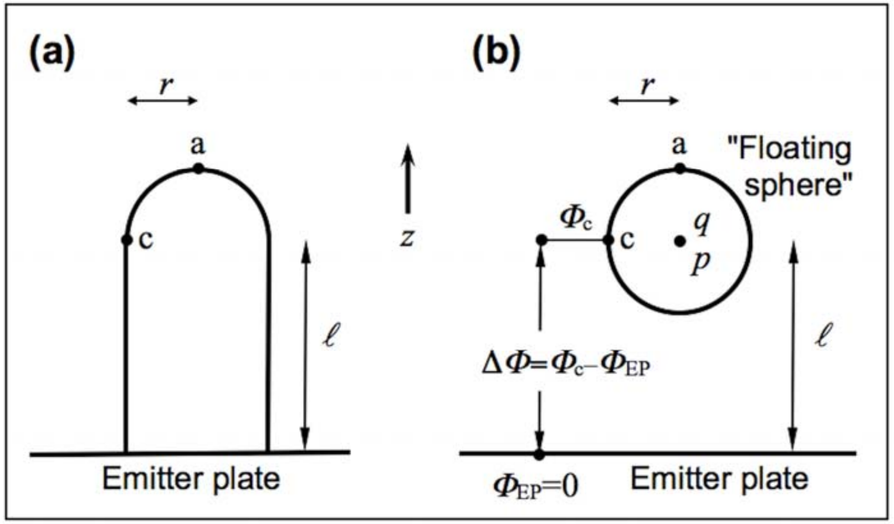

**In the simplest model version, the electrostatic potential $\Phi_c$ at "c" is given by the sum of the potential contributions:**

- **+ve contribution $|E_P|\ell$ due to the (–ve) applied macroscopic field $E_P$ ;**
- **–ve contribution $q/4\pi\varepsilon_0 r$ due to negative point charge $q$ ;**

**and the (–ve) field $E_a$ at point "a" is given approximately by $E_a = q/4\pi\varepsilon_0 r^2$ .**

# *FSEPP model for field electron emitter*






**The electrostatic potential $\Phi_c$ at "c" is given by the sum of:**

- **+ve contribution $|E_P|\ell$ due to the (–ve) applied macroscopic field $E_P$ ;**
- **–ve contribution $q/4\pi\varepsilon_0 r$ due to negative point charge $q$ ;**

**and the (–ve) field $E_a$ at point "a" is given approximately by $E_a = q/4\pi\varepsilon_0 r^2$ .**

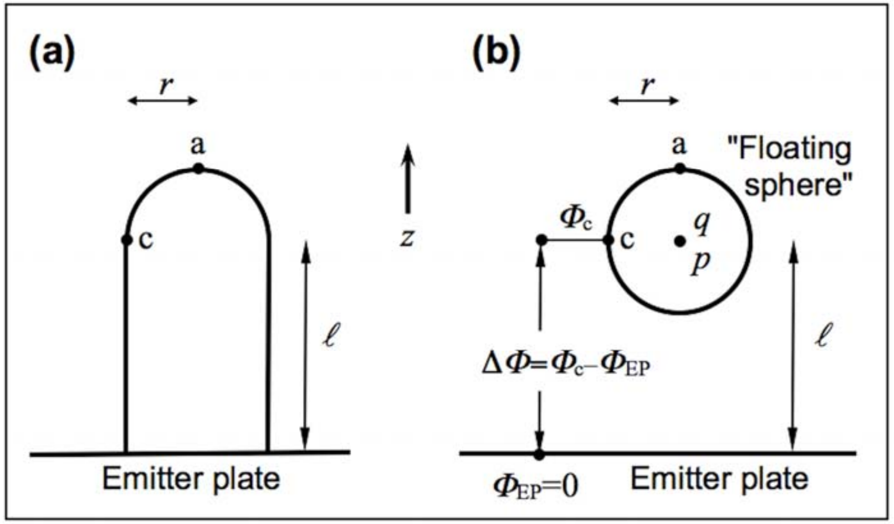
**To get $\Delta\Phi=0$, we put:** $q/4\pi\varepsilon_0 r = -|E_P|\ell = E_P\ell$ .

**Hence, the apex field $E_a$ is given by:** $E_a = q/4\pi\varepsilon_0 r^2 = E_P(\ell/r)$ .

**and the apex FEF has its "small-current value":** $\gamma_a^{sc} = E_a/E_P \approx (\ell/r)$ .

# *FSEPP model for voltage loss along post*






The electrostatic potential $\Phi_c$ at "c" is given by the sum of:

- +ve contribution $|E_P|\ell$ due to the (–ve) applied macroscopic field $E_P$ ;
- –ve contribution $q/4\pi\varepsilon_0 r$ due to negative point charge $q$ ;

and the (–ve) field $E_a$ at point "a" is given approximately by $E_a = q/4\pi\varepsilon_0 r^2$ .

When (+ve) voltage loss $V_d$ occurs along the post, then: $0 < \Delta\Phi < |E_P|\ell$ .

Hence the magnitude of $q$ must be reduced.

Hence the apex field and apex FEF are reduced in magnitude.

The apex FEF $\gamma_a$ is now given by: $\boxed{\gamma_a = [1 - V_d/(|E_P|\ell)]\ \gamma_a^{sc}}$ .

# *Voltage loss effects*



The **size** of a voltage loss can in principle be measured by contact-probe methods or by measuring total energy distributions.

In those cases where measurements exist, measured values suggest that non-ideality is more probably due to current dependence in the field enhancement factor, rather than series resistance effects.

This has important implications.

# What next ?

# *The value of $\lambda$*



**A major problem in FE science is that the value of $\lambda$ is unknown for any physically realistic barrier.**

**For the SN barrier, the "best guess in 2018" is that the value of the $\lambda^{SN}$ lies in the range**

$$0.005 < \lambda^{SN} < 14 ,$$

**but this could be an underestimate or an overestimate of the range of uncertainty.**

# *The origin of the $\lambda$-value problem*



**The origin of the $\lambda$-value problem lies in:**

- **(a) the use of smooth-surface conceptual models;**
- **(b) the failure (until recently) to formulate a theory of FN-type equations sufficiently general to allow the problem to be discussed;**
- **(c) the prolonged (90-year) failure to satisfactorily test the predictions of the smooth-surface models against experiment.**

**Smooth-surface models:**

- **disregard the existence of atoms;**
- **disregard the role of atomic wave-functions in transmission theory;**
- **assume the induced surface charge is located in an infinitesimally thin classical surface layer.**

**For real field emitters, these assumptions are wildly unrealistic.**

# Dealing with the $\lambda$-value problem

For the $\lambda$-value problem, there are two obvious solutions:

(a) prediction of $\lambda$, using much-improved atomic-level transmission theory;

(b) experimental measurement of $\lambda$ .

For **planar metal** surfaces, numerical quantum mechanics is just getting to the point where useful theoretical estimates of $\lambda^{SN}$ can be made, but different methods yield different values, so uncertainty still exists.

For sharply curved surfaces, the accurate prediction of $\lambda$ would be **intensely difficult**, probably beyond the existing boundaries of quantum mechanics. It may not be unreasonable to think in terms of a time-scale of **another 50-100 years** for its full solution.

# Dealing with the $\lambda$-value problem



**My solution to this situation is**

(1) Identify "mainstream emission theory".

(2) Identify and sort out any remaining problems in the theory.

(3) Make its presentation "properly scientific".

(4) Specify how to test the theory (which may not be straightforward), and specify how to measure $\lambda$.

(5) Advertise that the problem exists.

(6) Look for people and funding to do part or all of this testing/measurement.

# A strategic approach to FE theory

Elephant territory

Mainstream theory

[The first task is "to secure the perimeter"]

[& it seems best to deal with FNFE first]

Elephant territory

# *"Securing the perimeter"*

**Here are two examples of what is being done:**

**(a) checking the validity limits for the MG finite-temperature formula;**

**(b) establishing a standard method of extracting formal emission area.**

# *Re-investigation of the MG validity limits*



**The Murphy-Good zero-temperature FNFE equation for local ECD is**

$$J^{\mathrm{MG0}} \;=\; \mathrm{t_F}^{-2} a \phi^{-1} F^{2} \exp[-\mathrm{v_F} b \phi^{3/2}/F] \;.$$

**There is also a related finite-temperature MG FNFE equation, given by**

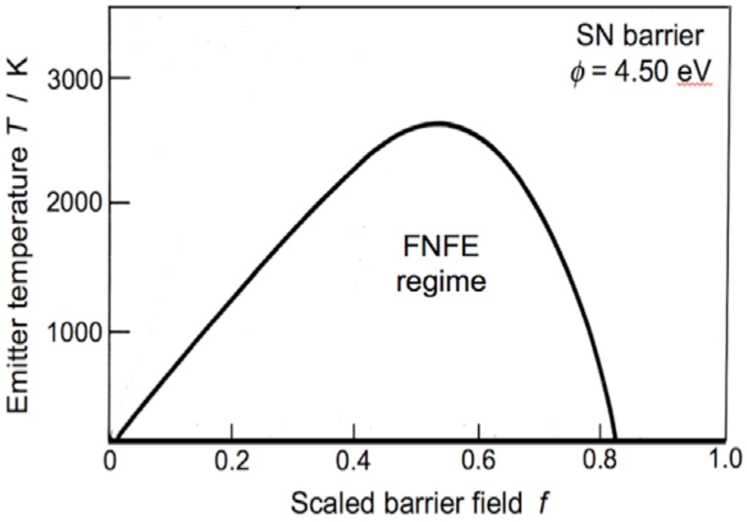
$$J^{\mathrm{MGT}} = \lambda_T J^{\mathrm{MG0}} ,$$

**where $\lambda_T$ is a temperature-dependent correction factor given by an equation of the form (details unimportant here)**

$$\lambda_T = (\pi p) / \sin(\pi p) \;.$$

# Re-investigation of the MG validity limits






$$\lambda_T = (\pi p) / \sin(\pi p) .$$

**It was known to MG that their derivation of the expression for $\lambda_T$ breaks down at sufficiently high fields and/or temperatures. Thus, they generated a validity-regime diagram, equivalent to that shown above.**

**One current project is to: (a) properly understand the physics behind this; and (b) re-calculate this diagram using modern methods.**

# Re-investigation of the MG validity limits



This is far less straightforward than it looks, because closer inspection reveals that a hierarchy of approximations were made in 1956. Thus, for planar emitters, the theory can in principle be done on at least six intellectual levels:

**Level 1:** The 1956 Murphy-Good approach.

**Level 2:** With improvements in their *mathematical* approximations.

**Level 3:** Numerical treatment in the Kemble semi-classical (SC) formalism.

**Level 4:** Numerical treatment in the Fröman & Fröman SC formalism.

**Level 5:** Numerical treatment using exact numerical solution of the Schrödinger equation.

**Level 6:** Quantum-mechanical treatments that take atomic-level effects and band-structure into account.

To begin with, there is a need to determine what MG actually did, and describe it in modern language, and also a need to give explicit proofs of some results where they gave inadequate detail.

# Standard method for extracting formal area



A second project relates to the development of a standard method for extracting formal emission area from current-voltage measurements.

At present, several different methods are in use that yield slightly different results for a given set of data.

I aim to identify which method is best or easiest, and implement it via a spreadsheet that can be downloaded. For example, a method has been proposed that relies on the formula

$$\text{Extracted formal area} = \Lambda_t^{SN}(\phi)\, R^{fit}\, (S^{fit})^2 ,$$

where $S^{fit}$ and $\ln\{R^{fit}\}$ are the slope and intercept of the line fitted to an experimental FN plot, and $\Lambda_t^{SN}(\phi)$ is an extraction parameter (for formal emission area) that can easily be calculated.

# *Dealing with the $\lambda$-value problem*



**My solution to this situation is**

**(1) Identify "mainstream emission theory".**

**(2) Identify and sort out any remaining problems in the theory.**

**(3) Make its presentation "properly scientific".**

**(4) Specify how to test the theory (which may not be straightforward), and specify how to measure $\lambda$.**

**(5) Advertise that the problem exists.**

**(6) Look for people and funding to do part or all of this testing/measurement.**

**Thanks for your attention**

# Immediately outstanding tasks



1. **Encourage all FE work to be presented using exclusively the International System of Quantities (i.e., abandon 1960s-style use of Gaussian system equations).**
2. **Encourage the LAFE experimental community to abandon use of the discredited original 1928 Fowler-Nordheim equation (or simplified versions of it), in favour of a more modern FE equation that takes at least the discoveries of the 1950s (and preferably later improvements) into account. And discourage the use of defective equations.**
3. **Encourage more uniform use of notation, e.g., encourage everyone to denote the principal SN barrier function by the symbol v (or *v*).**
4. **Encourage use of the Gauss variable (and of scaled barrier field), rather than the Nordheim parameter, when discussing the SN barrier.**
5. **Develop a single coherent approach to extracting formal emission area from Fowler-Nordheim plots (when emission is orthodox), and use it to replace the several slightly different methods currently being used.**

# Immediately outstanding tasks



6. Develop further the theory of FN-plot analysis for situations where the emission is "non-orthodox".
7. In connection with the "orthodoxy test" and related issues, investigate the extent of the "spurious results" problem in the literature, and the extent to which additional information can be extracted from published papers.
8. Find means of investigating experimentally whether the classical image potential energy is a satisfactory approximate model for the exchange-and-correlation interaction between a departing electron and the emitter.
9. Find means of investigating experimentally what is the actual power of local field in the pre-exponential of Fowler-Nordheim-type equations.
10. Find means of making experimentally-based estimates of the value of the characteristic pre-exponential correction factor $\lambda^{SN}$.
11. Investigate further the theory of transmission near the top of a SN barrier, and investigate discrepancies reported some years ago.

# Immediately outstanding tasks



12. Integrate into mainstream theory the more general "temperature-field" methods of calculating emission–current density developed by Jensen in recent years.
13. Establish improved methods of defining emission regimes.
14. Investigate further the issue of the validity of JWKB-type methods when the Schrödinger equation does not separate in Cartesian coordinates.
15. Investigate further the theory of field electron microscope resolution for very-small-radius emitters, where the apparent experimental ability to "resolve carbon bonds" is incompatible with existing theory.
16. Attempt to relate the theory of FE from carbon nanotubes more closely with mainstream FE theory.

# End