MATHEMATICAL MODELING OF TRIODE FIELD EMISSION SYSTEM WITH SHARP-EDGED CATHODE

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Abstract — Field emitters have great practical applications in such devices as electronic displays, cathode-ray light sources, X-ray sources, electron microscopes, etc. In this work the emission triode system based on an axially symmetrical sharp-edged field cathode on a plane substrate is under investigation. Anode is a plane, parallel to the substrate. The modulator is a circular diaphragm. The variable separation method is used to solve the boundary-value problem in cylindrical coordinates. To calculate the potential distribution an effect of the field cathode is simulated using the charged circular line. The electrostatic potential distribution is represented as Fourier-Bessel expansion. The unknown coefficients in the expansion are a solution of the linear algebraic equations with constant coefficients. The potential distribution is found throughout the triode system under study.

Keywords: field emitter, field emission, boundary-value problem, electron-optical system, potential distribution, Laplace/Poisson equation

Introduction

Vacuum devices of micro- and nanoelectronics based on field electron emission are used in many areas of scientific research, e.g., for electron microscopy, surface diagnostics [1,2]. Field cathodes may be of various forms [3]-[5]. In this parper the axially symmetrical triode system based on a sharp-edged field emitter is presented. The emitter substrate and anode are the planes. The modulator is a circular diaphragm. The diaphragm is in the plane parallel to the substrate and anode. Fig. 1 shows a schematic representation in the cylindrical coordinates (r, z) of the axially symmetrical triode system on the basis of the sharp-edged field emission cathode.

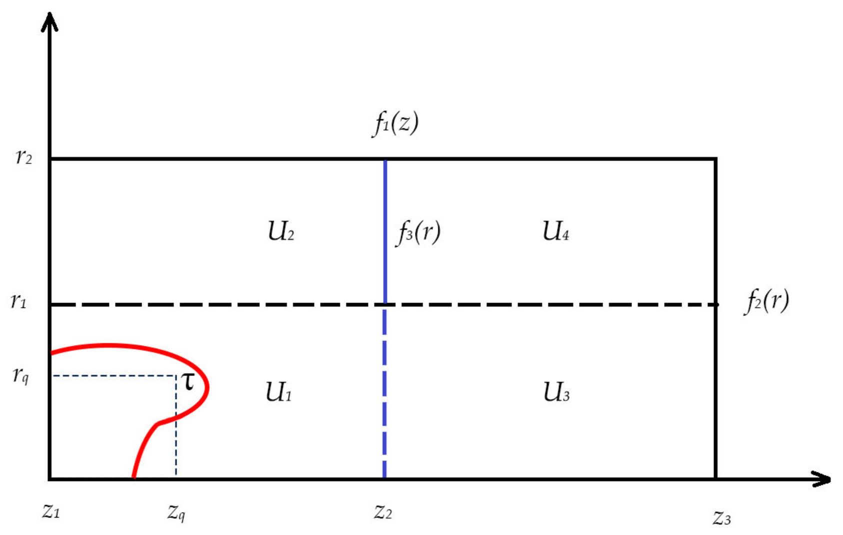

Figure 1: Schematic representation of the triode systems

The variable separation method is used to find the potential distribution electrostatic potential distribution $U(r,z)$. To obtain the emitter shape an effect of the field cathode is simulated using the charged circular line with the linear charge density τ so that the zero equipotential is the virtual cathode [6]-[9].
The parameters of the problem : $z = 0$ — the surface of the cathode substrate, $r = r_1$ — the modulator radius, $r = r_2$ — radius of the system region, (r_q, z_q) — the charged circular line τ position, $z = z_2$ $(r_1 \le r \le r_2)$ — the modulator surface, $z = z_3$ — the anode surface, $U(r, 0) = 0$ — the boundary condition at the substrate, $U(r_2, z) = f_1(r)$ — the boundary condition at the surface $r = r_2$, $U(r, z_2) = f_3(r)$ — the boundary condition at the modulator, $U(r, z_3) = f_2(r)$ — the boundary condition at the anode (Fig. 1).

Mathematical model

The electrostatic potential distribution $U(r,z)$ satisfies the Poisson equation with the boundary conditions

$$\begin{cases} \triangle\ U(r,z) = -\frac{1}{\varepsilon_0}\rho(r,z); \\ U(r,0) = 0; \\ U(r_2,z) = f_1(z), \quad z_1 \le z \le z_3; \\ U(r,z_3) = f_2(r), \quad 0 \le r \le r_2; \\ U(r,z_2) = f_3(r), \quad r_1 \le r \le r_2 \end{cases} \tag{1}$$

It is considered that the circular line τ forms a space charge distributed in a small volume

$$|r - r_q| < \delta_1, \quad |z - z_q| < \delta_2$$

with the constant volume charge density ρ so that

$$\tau = \lim_{\substack{\delta_1 \to 0 \\ \delta_2 \to 0}} 4\rho\delta_1\delta_2.$$

Then function $\rho(r,z)$ in the right side of the Poisson equation (1) can be written as

$$\rho(r,z) = \begin{cases} \rho, & |r - r_q| < \delta_1, \quad |z - z_q| < \delta_2, \\ 0, & |r - r_q| > \delta_1, \text{ or } |z - z_q| > \delta_2. \end{cases} \tag{2}$$

Solution of the boundary – value problem

To solve the boundary-value problem it can be generated four subregion **1** - $(0 \le r \le r_1,\ z_1 \le z \le z_2)$, **2** - $(r_1 \le r \le r_2,\ z_1 \le z \le z_2)$, **3** - $(0 \le r \le r_1,\ z_2 \le z \le z_3)$, **4** - $(r_1 \le r \le 2_2,\ z_2 \le z \le z_3)$.
Let

$$U(r,z) = \begin{cases} U_1(r,z),\ 0 \le r \le r_1, \quad z_1 \le z \le z_2; \\ U_2(r,z),\ r_1 \le r \le r_2, \quad z_1 \le z \le z_2; \\ U_3(r,z),\ 0 \le r \le r_1, \quad z_2 \le z \le z_3; \\ U_4(r,z),\ r_1 \le r \le 2_2, \quad z_2 \le z \le z_3. \end{cases} \tag{3}$$

The potential distribution (3) $U(r,z)$ can be represented as the Fourier-Bessel series:

$$\begin{aligned} U_1(r,z) = \sum_{m=0}^{\infty} b_m \frac{\sinh \frac{\gamma_m}{r_1}(z - z_1)}{\sinh \frac{\gamma_m}{r_1}(z_2 - z_1)} J_0(\frac{\gamma_m r}{r_1}) + + \sum_{n=0}^{\infty} a_n \frac{I_0\left(\frac{\pi n r}{(z_2 - z_1)}\right)}{I_0\left(\frac{\pi n r_1}{(z_2 - z_1)}\right)} \sin\frac{\pi n (z - z_1)}{z_2 - z_1} + \\ + \frac{2\tau r_q}{\varepsilon_1 r_1} \sum_{s=0}^{\infty} \frac{J_0(\frac{\gamma_s r_q}{r_1}) \sinh\frac{\gamma_s z_q}{r_1} \sinh(\frac{\gamma_s (z - z_3)}{r_1})}{\gamma_s J_1^2(\gamma_s) \sinh\frac{\gamma_s (z_2 - z_1)}{r_1}} J_0(\frac{\gamma_s r}{r_1}), \end{aligned} \tag{4}$$

$$\begin{aligned} U_2(r,z) = \sum_{m=1}^{\infty} d_m W_0(\lambda_m, r, r_2) \frac{\sinh \lambda_m (z - z_1)}{\sinh \lambda_m (z_2 - z_1)} + \\ + \sum_{n=1}^{\infty} \left(a_n \frac{\widetilde{W}_0(\frac{\pi n}{z_2 - z_1}, r, r_2)}{\widetilde{W}_0(\frac{\pi n}{z_2 - z_1}, r_1, r_2)} + g_n \frac{\widetilde{W}_0(\frac{\pi n}{z_2 - z_1}, r_1, r)}{\widetilde{W}_0(\frac{\pi n}{z_2 - z_1}, r_1, r_2)} \right) \sin\frac{\pi n (z - z_1)}{z_2 - z_1}, \end{aligned} \tag{5}$$

$$\begin{aligned} U_3(r,z) = \sum_{m=1}^{\infty} J_0(\frac{\gamma_m r}{r_1}) \left(b_m \frac{\sinh\frac{\gamma_m}{r_1}(z_3 - z)}{\sinh\frac{\gamma_m}{r_1}(z_3 - z_2)} + t_m \frac{\sinh\frac{\gamma_m}{r_1}(z - z_2)}{\sinh\frac{\gamma_m}{r_1}(z_3 - z_2)} \right) + \\ + \sum_{k=0}^{\infty} c_k \frac{I_0\left(\frac{\pi k r}{(z_3 - z_2)}\right)}{I_0\left(\frac{\pi k r_1}{(z_3 - z_2)}\right)} \sin\frac{\pi k (z - z_2)}{z_3 - z_2}, \end{aligned} \tag{6}$$

$$\begin{aligned} U_4(r,z) = \sum_{m=0}^{\infty} W_0(\lambda_m, r, r_2) \left(d_m \frac{\sinh\lambda_m (z_3 - z)}{\sinh\lambda_m (z_3 - z_2)} + s_m \frac{\sinh\lambda_m (z - z_2)}{\sinh\lambda_m (z_3 - z_2)} \right) + \\ + \sum_{k=0}^{\infty} \left(c_k \frac{\widetilde{W}_0(\frac{\pi k}{z_3 - z_2}, r, r_2)}{\widetilde{W}_0(\frac{\pi k}{z_3 - z_2}, r_1, r_2)} + h_k \frac{\widetilde{W}_0(\frac{\pi k}{z_3 - z_2}, r_1, r)}{\widetilde{W}_0(\frac{\pi k}{z_3 - z_2}, r_1, r_2)} \right) \sin\frac{\pi k (z - z_2)}{z_3 - z_2}, \end{aligned} \tag{7}$$

where

$W_0\,(\lambda, x, y) = J_0\,(\lambda x)\ Y_0\,(\lambda y) - J_0\,(\lambda y)\ Y_0\,(\lambda x)$,

$\bar{W}_0\,(\lambda, x, y) = I_0\,(\lambda x)\ K_0\,(\lambda y) - I_0\,(\lambda y)\ K_0\,(\lambda x)$,

$J_0\,(\lambda r)$, $Y_0\,(\lambda r)$ — Bessel functions of the first and second kind accordingly,

$I_0\,(\lambda r)$, $K_0\,(\lambda r)$ — modified Bessel functions of the first and second kind accordingly,

γ_n — the zeros of Bessel functions $J_0\,(\gamma_n) = 0$,

η_n — the zeros of Bessel functions linear combination $W_0\,(\eta_n, R_1, R_2) = 0$.

The Fourier-Bessel series coefficients d_m, g_n, t_m, s_m, h_k of the potential distributions (4)–(7) are the eigenfunction expansion coefficients of the boundary functions $f_1(z)$, $f_2(z)$, $f_3(z)$ (1).
The boundary conditions on the planes $z = z_2$ and $r = r_1$ of the potential distributions (4)–(7) are used to find the unknown coefficients a_n, b_m, c_k:

$$\left.\frac{\partial U_1(r,z)}{\partial r}\right|_{r=r_1} = \left.\frac{\partial U_2(r,z)}{\partial r}\right|_{r=r_1}, \qquad z_1 \le z \le z_2, \tag{8}$$

$$\left.\frac{\partial U_3(r,z)}{\partial r}\right|_{r=r_1} = \left.\frac{\partial U_4(r,z)}{\partial r}\right|_{r=r_1}, \qquad z_2 \le z \le z_3. \tag{9}$$

$$\left.\frac{\partial U_1(r,z)}{\partial r}\right|_{z=z_2} = \left.\frac{\partial U_3(r,z)}{\partial r}\right|_{z=z_2}, \qquad 0 \le r \le r_1. \tag{10}$$

In this article a triode system on the basis of a sharp-edged field cathode as the axially symmetrical electron-optical systems is under investigation. The field emitter substrate anode are the planes. The modulator is a circular diaphragm. The potential of the cathode and substrate is equal the zero. The variable separation method is used in cylindrical coordinates (r, z) to solve the boundary-value problem (1)–(2). The electrostatic potential distributions (3) are represented as the Fourier-Bessel series (4)–(7). Some of the Fourier-Bessel series coefficients are the expansion coefficients of the boundary functions. The boundary conditions (8)–(10) of the potential distributions can be used to find the unknown coefficients as the solution of the system of linear algebraic equation with the constant coefficients.

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