FOUR-PARAMETER MODEL FOR A FEE SIGNAL

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Introduction

Devices based on the phenomenon of field electron emission (FEE) continue to be considered as extremely promising [1]. Their advantages and limitations are well known. Partial neutralization of deficiencies can be achieved by using carbon materials, diamond, silicon carbide, cadmium sulfide and some others.

The basic law for FEE describing remains the Fowler – Nordheim formula for the relation of the current density j with the electric field strength E:

$$j = aE^2 \exp\left[-\frac{b}{E}\right], \quad a = \frac{e^3}{16\pi^2\hbar\Phi t^2(\tau)}, \quad b = \frac{4\sqrt{2m}}{3\hbar e}\Phi^{\frac{3}{2}} v(\tau), \qquad (1)$$

$\tau = \frac{e}{\Phi}\sqrt{\frac{eE}{4\pi\varepsilon_0}}$; e is elementary charge, m is electron mass, $\hbar$ is reduced Planck constant, ε_0 is electric constant, $t(\tau)$ and $v(\tau)$ are Nordheim functions. The dependence of $\lg(j/E^2)$ on $1/E$ is linear. By the straight line slope you can judge the work function value Φ.

In integral representation (when replacing j by I and E by V), the formula **(1)** is generally preserved. The new parameters of A and B additionally depend on the work function distribution over the surface, on the surface geometry. However, it is difficult to imagine that a two-parameter model is capable of providing an exhaustive description of the source. It is quite clear that the experiment significantly departed from the premises laid down in the Fowler–Nordheim theory. Moreover, deviations from the linear dependence of the expression $\lg(I/V^2)$ as a function of $1/V$ (see, for example, [2, 3]) are not analyzed using other models.

Possible and real deviations from the Fowler — Nordheim law are interpreted differently. Current reduction is explained by presence of a space charge. An increase in the current is promoted by growth of the emission surface and auxiliary events, for example, the appearance of an internal field. A change in the cathode state can lead to multimodality of the electron energy distribution [4]. In some cases, hysteresis is observed [5].

It is clear that in addition to voltage, other factors can also influence the response, for example, temperature. How significant can unaccounted circumstances of an experiment be? You can try to answer this question in terms of a regression analysis. Consider a four-parameter model based on theoretical investigations.

Regression approach

Let there be a set of measurements $\{x_i, \tilde{y}_i\}$, where $i = \overline{1, N}$. The factor values $\mathbf{x} = (x_1, \ldots, x_N)$ must be determined exactly, and the responses $\tilde{\mathbf{y}} = (\tilde{y}_1, \ldots, \tilde{y}_N)$ contain errors. We will write:

$$\tilde{y}_i = y_i + \varepsilon_i = f^*(x_i; \mathbf{q}^*) + \varepsilon_i, \quad x = V/V_0, \quad y = I/I_0.$$

Here f^* is the estimated unknown response function, $\mathbf{q}^* = (q_1^*, q_2^*, \ldots, q_p^*)$ is the set of unknown parameters, ε_i are measurement errors (noise). The values V_0 and I_0 allow to work with dimensionless quantities.

Imagine the response in the form:

$$I/I_0 = f(x; \mathbf{q}) = Ax^{2+\eta} \exp[-B/x] \exp\left[-C/x^2\right]. \qquad (2)$$

The coefficient C was proposed in [6]. The addition of the η parameter is dictated by many theoretical premises (for example, [7, 8, 9]). The three-parameter regression model is considered in [10]. If $C = \eta = 0$, we are dealing with the classical case. Two-parameter regression considered in [11]. Within the model **(2)** framework, it is proposed to check the statistical significance of the coefficients C and η [12]. If the hypothesis about the significance of a any parameter is rejected, you can lower the dimension p.

For parameters $\mathbf{q}^*$ it is necessary to obtain an optimal estimate $\hat{\mathbf{q}}$. The regression function $f(x; \mathbf{q})$ is also an estimate of the true dependence $f^*(x; \mathbf{q})$. As a result, we can write

$$\tilde{y}_i = f(x_i; \hat{\mathbf{q}}) + \hat{e}_i, \quad J(\mathbf{x}, \tilde{\mathbf{y}}; \hat{\mathbf{q}}) = \min_{\mathbf{q} \in Q} J(\mathbf{x}, \tilde{\mathbf{y}}; \mathbf{q}), \qquad (3)$$

where $Q = \left\{(A, B, C, \eta) : A > 0, B > 0, C \in \mathbb{R}, \eta > -2\right\} \subseteq \mathbb{R}^p$ is the parameter definition area, $J(\mathbf{x}, \tilde{\mathbf{y}}; \mathbf{q})$ is some quality functional, $\hat{e}_i$ are residuals [12].

Linearization (Fowler—Nordheim coordinates)

In our case, the function $f(x, \mathbf{q})$ is nonlinear in parameters. We carry out the traditional transformations (Fowler—Nordheim coordinates):

$$X = 1/x, \quad Y = \lg\left[y/x^2\right]. \qquad (4)$$

This allows us to obtain new observations instead of **(3)** in the form

$$\tilde{Y}_i = \varphi(X_i; \hat{\vartheta}) + \hat{e}_i = \sum_{j=1}^{p} \hat{\vartheta}_j g_j(X_i) + \hat{e}_i, \qquad (5)$$

where the area $\Theta \subseteq \mathbb{R}^p$ is transformation of Q via **(4)**. When considering **(5)** the structure of J may change, but may remain the same — it depends on the researcher. The transformed response model $Y(X)$ can rightly be called linearized in parameters. Note that the residuals $\hat{e}_i$ in **(5)** have changed due to the conversion.

We write out the new notations:

$$\vartheta_1 = \lg A, \quad \vartheta_2 = -B/\ln 10, \quad \vartheta_3 = -C/\ln 10, \quad \vartheta_4 = -\eta;$$

$$g_1(X) = 1, \quad g_2(X) = X, \quad g_3(X) = X^2, \quad g_4(X) = \lg X.$$

The **(5)** model is most usable if the least squares method is used. This approach has a lot of statistical benefits [12]. For their demonstration, the following requirements must be met:

- the residuals $\hat{e}_i$ must be independent random variables;
- the residuals $\hat{e}_i$ must belong to the normal distribution law, and $\mathsf{E}\,\hat{e}_i = 0$, $\mathsf{D}\,\hat{e}_i = \mathrm{const}$ for any i.

Further, we will follow to the study of **(2)** in terms of mathematical modeling. Thus, noise will be imported into the signal using a pseudo random number generator. This will allow us to consider the residuals independence as realized. The second condition — homoskedasticity — is a serious question. Moreover, to check it, it is desirable to have repeated measurements.

Errors

Contrary to requirements regarding factor values, voltage measurements may also contain errors. As a result, we have:

$$\tilde{V}_i = V_i + V_0\varepsilon_{V,i}, \quad \tilde{I}_i = I_i + I_0\varepsilon_{I,i}, \quad \tilde{x}_i = x_i + \varepsilon_{V,i}, \quad \tilde{y}_i = y_i + \varepsilon_{I,i}. \qquad (6)$$

The response will be more accurate as:

$$\tilde{y}_i = f^*(x_i + \varepsilon_{V,i}; \mathbf{q}^*) + \varepsilon_{I,i}, \quad i = \overline{1, N}.$$

For the regression function **(2)**, voltage errors will contribute:

$$f(x + \varepsilon_V; \mathbf{q}) = f(x; \mathbf{q})\left[1 + \left(\frac{2+\eta}{x} + \frac{B}{x^2} + \frac{2C}{x^3}\right)\varepsilon_V + o(\varepsilon_V)\right]. \qquad (7)$$

The resulting noise will take the form:

$$\varepsilon = \varepsilon_I + f(x; \mathbf{q})\left[\left(\frac{2+\eta}{x} + \frac{B}{x^2} + \frac{2C}{x^3}\right)\varepsilon_V + o(\varepsilon_V)\right].$$

The denominators in **(7)** are a possible cause of residuals heteroscedasticity. This is a common property, which is true for the original Fowler—Nordheim model too.

Errors **(6)** will be reflected in the linearized response **(5)**:

$$\tilde{Y} = \lg\left[\frac{f(x, \mathbf{q}) + \varepsilon}{x^2}\right] = \varphi(X; \vartheta) + \frac{\varepsilon}{\lg 10 f(x, \mathbf{q})} + o(\varepsilon).$$

If we consider a progressive model of noise with a normal distribution law, then we can write $\varepsilon_{V,i} = x_i\delta_V\varepsilon_i'$, $\varepsilon_{I,i} = y_i\delta_I\varepsilon_i''$, where ε_i' and ε_i'' are independent values of the standard Gaussian random variable, δ_V and δ_I are responsible for the signal-to-noise ratio. Then

$$\tilde{Y} = \varphi(X; \vartheta) + \frac{1}{\lg 10}\left[\left(2 + \eta + \frac{B}{x} + \frac{2C}{x^2}\right)\delta_V\varepsilon' + \delta_I\varepsilon''\right] + o(\delta_V) + o(\delta_I).$$

It can be seen that only current errors allow us to rely on homoscedasticity of $\hat{e}_i$. Voltage errors can lead to heteroscedasticity, which will require additional approaches to statistical analysis and estimates [13, 14] — fig. 1.

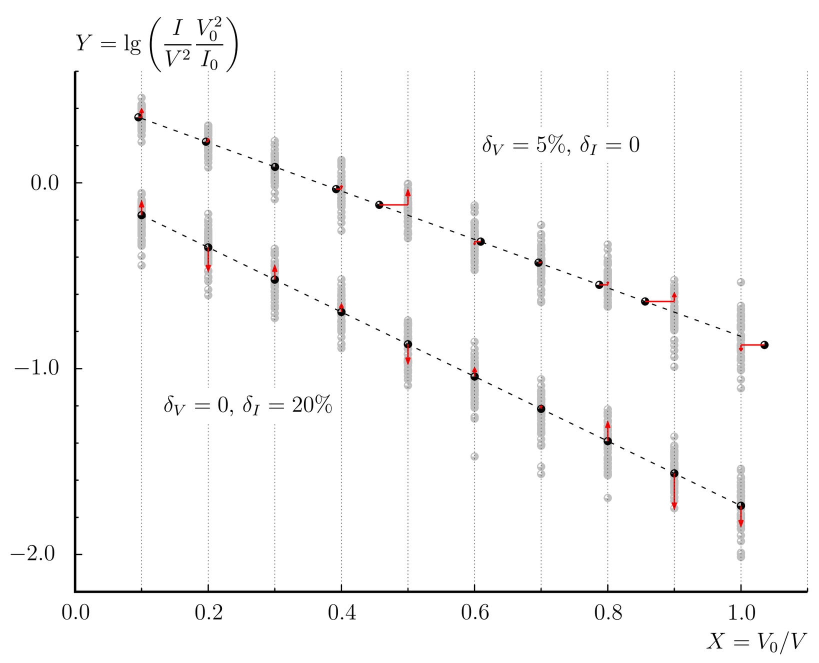

Рис. 1: *Errors for the Fowler–Nordheim model*

Some results

Fig. 1 shows the simulation results obtained with the following values: $V_0 = 1$ V, $I_0 = 1$ A, $V_{\min} = 1$ V, $V_{\max} = 10$ V. The behavior of the $I-V$ curve and errors for the Fowler–Nordheim model is shown ($A^* = 2, 3$, $B^* = 3, 4$, $N = 10$). 50 repeating current values are given so that homoskecasticity and heteroscedasticity can be observed visually. Also a mechanism for errors generating in measuring voltage and current is presented.

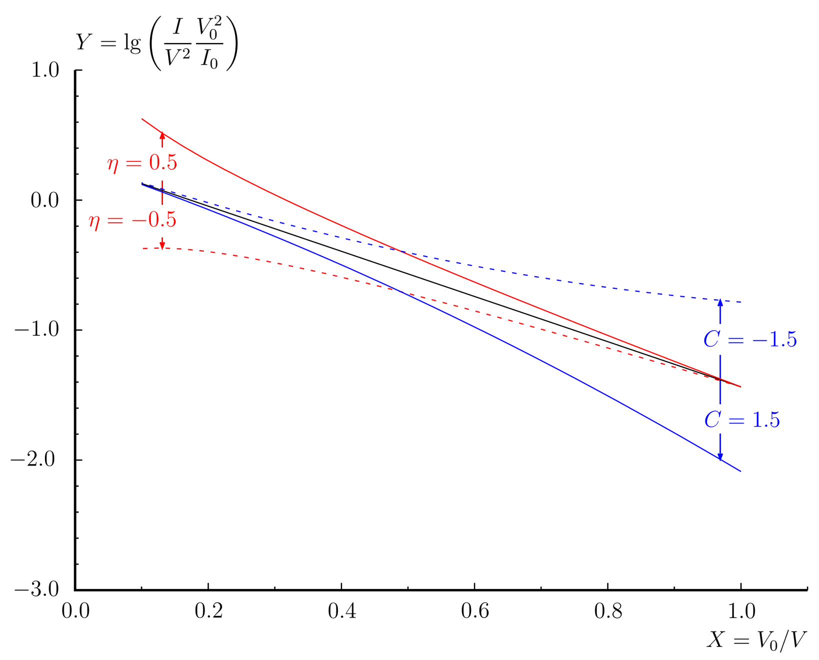

Рис. 2: *The influence of the coefficients C and η*

Fig. 2 shows the influence of the coefficients C and η on the $I-V$ curve. Here $A^* = 2$, $B^* = 4$. It can be seen that it is possible to take into account deviations from the Fowler–Nordheim law for both low and high voltage values.

Finally, consider the possibility of taking into account hysteresis. For a closed curve, there are points x such that

$$A_1 x^{2+\eta_1} \exp\left[-B_1\frac{1}{x}\right] \exp\left[-C_1\frac{1}{x^2}\right] = A_2 x^{2+\eta_2} \exp\left[-B_2\frac{1}{x}\right] \exp\left[-C_2\frac{1}{x^2}\right].$$

Since the coefficient B is most closely related to the work function, we require that $B_2 = B_1$. Consider two points $\bar{x} \neq \bar{\bar{x}}$, moreover $\bar{x} = 1$. Then

$$\ln\frac{A_1}{A_2} = (C_1 - C_2), \quad \ln\frac{A_1}{A_2} + (\eta_1 - \eta_2)\ln\bar{\bar{x}} = (C_1 - C_2)\frac{1}{\bar{\bar{x}}^2}.$$

Coefficient A is responsible for the curve vertical displacement in the Fowler–Nordheim coordinates. Let $\ln\frac{A_1}{A_2} = \gamma$. Then for the given coefficient C_1 it follows from the equation for $\bar{x}$ that:

$$C_2 = C_1 - \gamma.$$

Transforming the equation for $\bar{\bar{x}}$, we get that for a given η_1

$$\eta_2 = \frac{1}{\ln\bar{\bar{x}}}\left(\eta_1\ln\bar{\bar{x}} + \gamma\left(1 - \frac{1}{\bar{\bar{x}}^2}\right)\right).$$

Fig. 3 represents the result for the following values: $A_1 = 2$, $B_1 = 4$, $C_1 = -1.5$, $\eta = 0$ (curve 1). It was prescribed $B_2 = B_1 = 4$, $A_2 = 2A_1 = 4$. This led to values $C_2 = -0.114$, $\eta_2 = -0.596$ (curve 2).

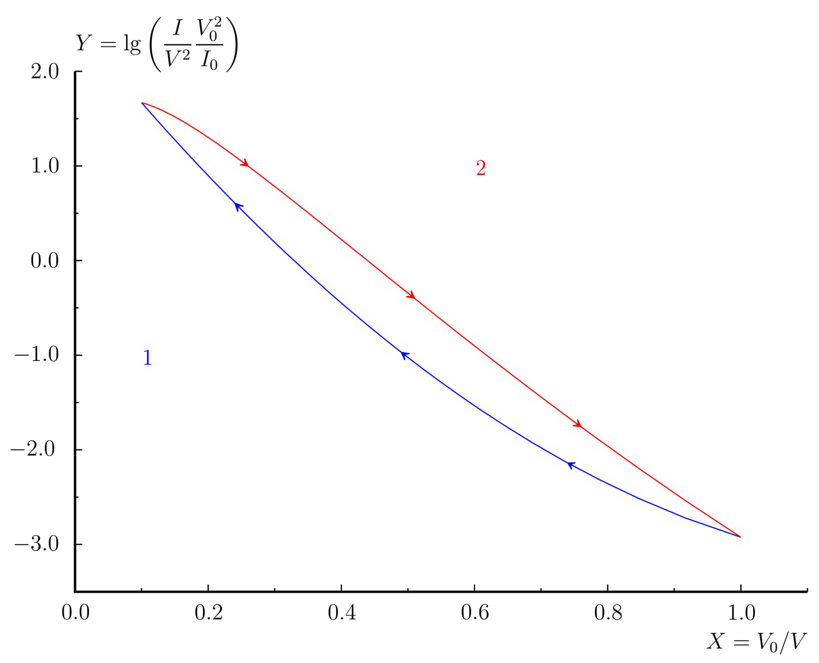

Рис. 3: *Description of the hysteresis in terms of the four-parameter model*

Conclusion

The paper considers the four-parameter regression model for the FEE signal describing. There are theoretical preconditions for using such a regression function. In terms of mathematical modeling, the behavior of the noise generated by both voltage and current errors in measurements is shown. The proposed approach allows one to take into account the $I-V$ curve deviations from the straight line in the Fowler–Nordheim coordinates at any voltage. A hysteresis response is also possible for describing.

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