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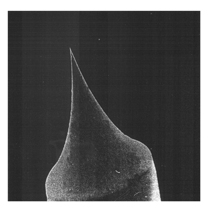

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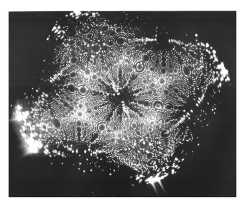

UNIVERSITY OF SURREY

USING FOWLER-NORDHEIM OR MURPHY-GOOD PLOTS TO MEASURE CHARACTERISTIC VALUES OF FIELD AND SCALED FIELD

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SUMMARY

This Poster aims to remind people that IDEAL Fowler-Nordheim and Murphy-Good plots (based on conventional parameters or on scaled parameters) can be used to measure characteristic values of local barrier field F_C and the related scaled field f_C , and discusses related theory. It shows why the plots need to be ideal. It also notes work that assesses the accuracy of these methods. An application to past low-macroscopic-field (LMF) electron emission measurements from carbon suggests that an alternative explanation of the LMF phenomenon is that carbon emitters can exhibit anomalously high field enhancement factors. This conclusion is relevant to discussions of the origin of electrical breakdown in poor vacuum conditions.

1. THE ROLE OF VOLTAGE CONVERSION LENGTH

When interpreting measured **field electron emission (FE)** current-voltage $I_m(V_m)$ characteristics, it is now widely recognized that both emission-physics aspects and circuit-theory aspects of FE devices/systems must be considered. In older work, e.g. [1], the relation between V_m and the emitter's characteristic local barrier field F_C may be written

$$F_C = \beta_V V_m . \tag{1}$$

where β_V (often written as β) has the units m^{-1}, or equivalent. However, to avoid confusion with modern usage of the symbol β to represent dimensionless field enhancement factors, I now prefer to write eq. (1) in the equivalent form

$$F_C = V_m/\zeta_C , \tag{2}$$

where ζ_C [$=1/\beta_V$] is a **characteristic voltage conversion length (VCL)** defined by eq. (2). VCLs are characterization parameters, not physical lengths.

In modelling, it is usually convenient to take the characteristic location "C" to be at the emitter apex, but for some real emitters the location "C" may best be taken elsewhere, for example at the location where the local emission current density (ECD) is highest.

The derivation of the 1956 Murphy-Good FE equation [2,3] assumes that tunnelling takes place through a **Schottky-Nordheim (SN)** ("planar image rounded") barrier.

In the so-called **Extended Murphy-Good (EMG) FE equation** [4], the pre-exponential factor t_F^{-2} that appears in the zero-temperature version of the 1956 MG FE equation is replaced by a correction factor (or **knowledge uncertainty factor**) λ, and a parameter A_f^{SN} (called the **formal area for the SN barrier**) is used. Assuming that there is no leakage current, the EMG equation for $I_m(V_m)$ is [4]:

$$I_m = A_f^{SN} a\phi^{-1}(V_m/\zeta_C)^2 \exp[-\nu_F b\phi^{3/2}\zeta_C/V_m] , \tag{3}$$

where a and b are the FN constants [5] and ν_F is a particular value (appropriate to a barrier defined by ϕ and F_C) of the **FE special mathematical function** $\nu(x)$, expressed here as a function of the Gauss variable x.

2. FOWLER-NORDHEIM PLOTS USING CONVENTIONAL PARMETERS

In **natural FN coordinates**, eq. (3) becomes

$$\ln\{I_m/V_m^2\} = \ln\{A_f^{SN} a\phi^{-1}\zeta_C^{-2}\} - \nu_F b\phi^{3/2}\zeta_C/V_m . \tag{4}$$

If the dependence on V_m of the term $\ln\{A_f^{SN} a\phi^{-1}\zeta_C^{-2}\}$ is weak, then the **local slope $S(V_m^{-1})$** of the theoretical FN plot is

$$S(V_m^{-1}) \equiv \mathrm{d}\ln\{I_m/V_m^2\}/\mathrm{d}(V_m^{-1}) = -b\phi^{3/2}\,\mathrm{d}[\nu_F\zeta_C V_m^{-1}]/\mathrm{d}V_m^{-1} . \tag{5}$$

This reduces to the form

$$S(V_m^{-1}) = -b\phi^{3/2}\,[s\zeta_C - \nu_F V_m \mathrm{d}\zeta_C/\mathrm{d}V_m] , \tag{6}$$

where s is the **slope correction function** as usually defined (e.g., [3]).

Since s is known to have only a weak dependence on V_m^{-1}, it follows that the condition for the FN plot to be "only slightly curved" is that we should have

$$\mathrm{d}\zeta_C/\mathrm{d}V_m \approx 0 , \tag{7}$$

which means that ζ_C must be constant and independent of V_m. An FE device/system for which this is true (or effectively true), and corresponding FN plots, are termed **ideal**.

If an FN plot is ideal, then it can be used to measure characteristic fields. In this case, ζ_C can be found from

$$\zeta_C = -S^{fit}/s_t b\phi^{3/2} , \tag{8}$$

where S^{fit} is the slope of a straight line fitted to an experimental FN plot, and s_t is the **fitting value** of s. It is usually adequate to approximate $s_t \approx 0.95$. Once ζ_C is known, measured V_m-values can be converted to F_C-values by using eq. (2). Non-ideal FN plots may be excluded by using an **orthodoxy test** [6].

Procedures equivalent to these can also be implemented using **Murphy-Good (MG) plots** [4], in which case s_t is replaced by unity.

3. FOWLER-NORDHEIM PLOTS USING SCALED PARMETERS

Equivalent results can be obtained if the exponent in (3), which represents the **Fermi-level value D_F of tunnelling probability**, is written in **scaled form** as

$$D_F = \exp[-\nu_F\eta/f_C] = \exp[-\nu_F(bc^2\phi^{-1/2})(V_{mR}/V_m)] . \tag{9}$$

Here, the **Schottky constant c** $\equiv (e^3/4\pi\varepsilon_0)^{1/2}$, where e is the elementary charge and ε_0 the vacuum electric permittivity, and f_C is the **characteristic scaled (barrier) field** defined by:

$$f_C \equiv V_m/V_{mR} = F_C/F_R \equiv c^2\phi^{-2}F_C . \tag{10}$$

Here, η [$\equiv bc^2\phi^{-1/2}$] is a **scaling parameter**. F_R [$\equiv c^{-2}\phi^2$] is the **reference field**, and V_{mR} the **reference measured voltage**, at which the top of the barrier, at the characteristic location "C", is pulled down to the Fermi level.

By arguments similar to Section 3, **ζ_C=constant** implies **V_{mR}=constant**, and (for ideal devices/systems and related FN plots) V_{mR} can be obtained from

$$V_{mR} = -S^{fit}/s_t\eta = -S^{fit}/s_t bc^2\phi^{-1/2} . \tag{11}$$

Thus, measured V_m-values can be converted to f_C-values by using (10).

A source of error in these procedures is that the relevant local work function ϕ may not be well known. Since ζ_C depends on ϕ as $\phi^{-3/2}$, but V_{mR} depends on ϕ as $\phi^{1/2}$, it follows that values of scaled field f_C extracted from FN plots via (11) and (10) are more accurate than values of barrier field F_C extracted via (2).

4. COMPARISONS WITH EXPERIMENT

Two routes to comparisons with experiment are currently available. The first is the old (1953) comparison made by Dyke and Trolan [7], using their emitter X89. This concluded that there was agreement between FE-based methods and electron-microscopy-based methods for field measurement, to within around 20%.

The second is a modern (2019) electron-interference-based method [8] which deduced a field 2.93 V/nm , for an operating tungsten emitter. For comparison, an emitter with ϕ= 4.50 eV, operating at f_C= 0.25 (a typical mid-range value) would have F_C= 3.5 V/nm. The level of agreement is very encouraging.

5. LOW MACROSCOPIC FIELD EMISSION FROM CARBON

It is well established (e.g., [9,10]) that emission from large-area carbon emitters sometimes occurs at very low macroscopic fields. Special emission mechanisms (e.g., resonance tunneling) have been suggested. Related papers (e.g., [10-12]) contain FN plots, and these have been used to find the range of local fields actually present. Results are:

Origin of data	Barrier-field range
[10], Fig. 10 (lowest curve)	3.3 V/nm $< F_C <$ 5.8 V/nm
[11], Fig. 8.13:	2.9 V/nm $< F_C <$ 5.1 V/nm
[12], Fig. 2, curve 1:	4.9 V/nm $< F_C <$ 7.4 V/nm

Whilst special mechanisms may also operate, these results strongly suggest that the primary reason for LMF emission from carbon emitters is that they exhibit **anomalously high field enhancement factors**. This conclusion may be of interest to discussions of the origin of vacuum breakdown effects in some high-gradient accelerators and high-vacuum devices. More generally, using FN or MG plots to measure local fields may be of interest in various FE contexts.

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