USING FOWLER-NORDHEIM PLOT FOR THE COMPARISON OF ELECTRON EMISSION EFFICIENCY OF FIELD (FE) AND THERMAL EMISSION (TE) CATHODES

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7th ITG International Vacuum Electronics Workshop (IVEW) 2020
and
13th International Vacuum Electron Sources Conference (IVeSC) 2020

planned for Physikzentrum Bad Honnef (PBH), Bad Honnef, Germany
(virtual conference/Thales AVS France, Paris),
May 26-29, 2020,
Lecture IVEW23, May 26, 2020, 15:30 CEST (3:30 p.m.)

Knapp, Wolfram: „Using Fowler-Nordheim plot for the comparison of electron emission efficiency of field (FE) and thermal emission (TE) cathodes. 7th ITG International Vacuum Electronics Workshop (IVEW) 2020 and 13th International Vacuum Electron Sources Conference (IV*e*SC) 2020 , planned for Physikzentrum Bad Honnef (PBH), Germany (virtual conference/Thales AVS France), May 26-29, 2020, Lecture IVEW23, May 26, 2020, 3:30 pm MESZ.

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OUTLINE:

1. **Introduction**
2. **Motivation**
3. **Definition of Electron Emission Efficiency (3E)**
4. **Fowler Nordheim (FN) plot**
   - Standard use of the FN plot for field emission analysis
   - Extended use of the FN plot for comparison of electron emission efficiency
5. **Comparison of field (FE) and thermal (TE) cathodes**
6. **Conclusion and outlook**

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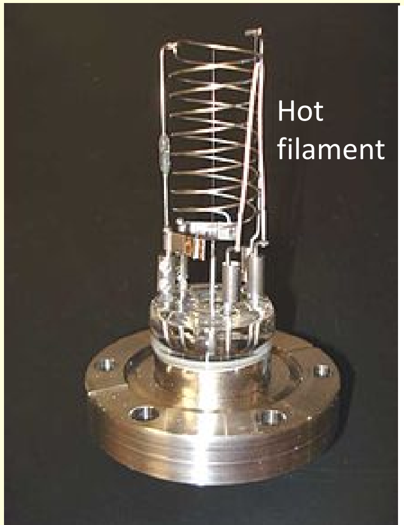
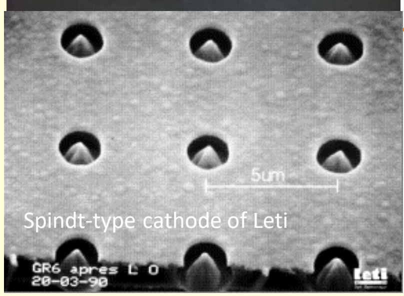
Motivation: Ionization Gauges

Typical application of cathodes with **1 mA** emission current and the question:

What is better?





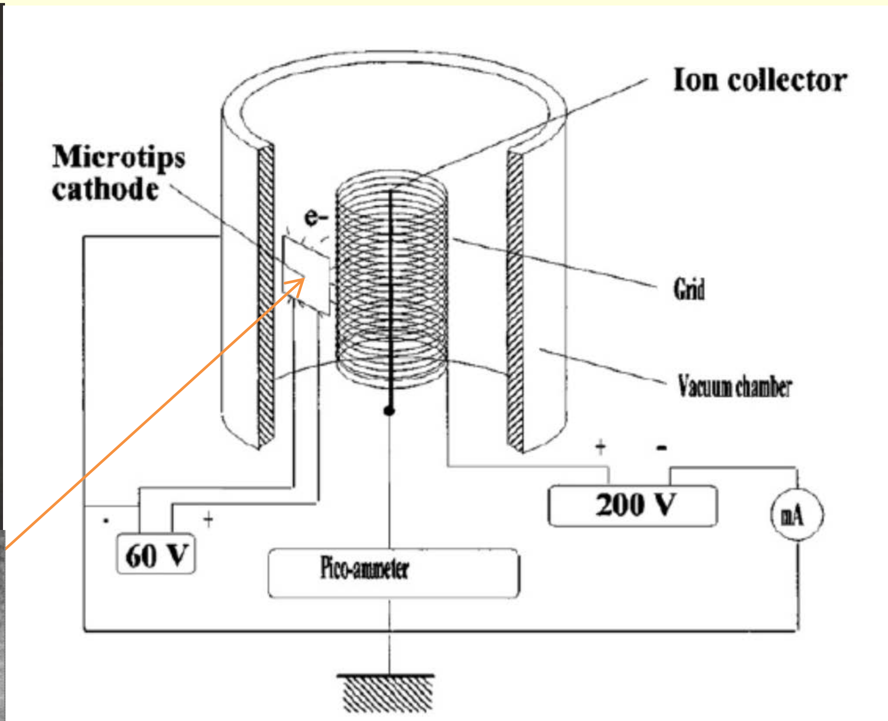
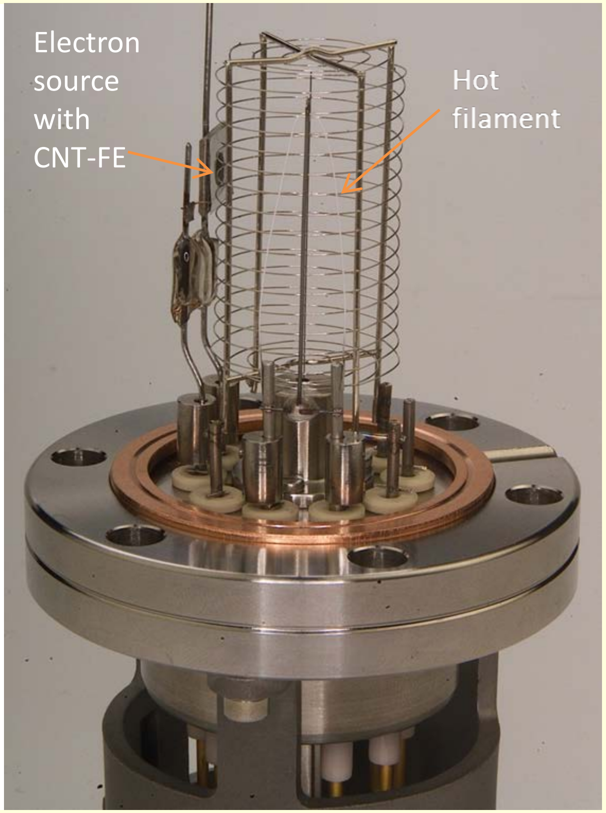


Fig. 7. BAG with FEA after Baptist. Reprinted with permission from Ref. [24]. Copyright 1996, American Vacuum Society.



Ionization gauge with thermal (hot filament) or field emitter cathodes (Spindt-type [1], CNT-FE [2], SS [3])

Literature:

[1] R. Baptist, C. Bieth, C. Py: Bayard–Alpert vacuum gauge with microtips, J. Vac. Sci. Techn. B, 14 (1996) 2119-2125.

[2] W. Knapp, D. Schleussner, M. Wüest: Investigation of ionization gauges with carbon nanotube (CNT) field-emitter cathodes. J. of Physics: Conf. Series 100 (2008) 092007.

[3] W. Knapp, M. Wüest: "Vacuum measuring gauge", patent WO 2006/094687, priority data 04.03.2005.

Electron Emission Efficiency - Definition

Electron Emission Efficiency (3E): $$v_{3E} = \frac{emission\ current\ I_E}{spent\ electrical\ power\ P_{el}} = \frac{I_E}{P_{el}} \quad [V^{-1}]$$

The relation I_E/P_{el}, easy to measure (e.g. for TE is $P_{el} = P_{heat}$), is therefore a simple criterion reflecting the electron emission efficiency, **independent of the kind of electron emission mechanism** (Tab. 1):

Table 1. Comparison of the electrical power P_{el} for different typical cathode types (without significant loss of power)

Cathode:	Cold Field Emitter FE	Thermionic TE (Hot Filament)	Dispenser-C. (with Schottky effect)	Plasma-C. or Electron Source
$P_{el} =$	$P_{extr} = I_E \cdot U_{extr}$	$P_{heat} = I_{heat} \cdot U_{heat}$ $(= I_f \cdot U_f)$	$P_{heat}(+P_{extr})$ cf. TE- (and FE-) Cathodes	$P_{pl} = I_{pl} \cdot U_{pl}$ $[+P_{extr}]$

A **special case** is the emission efficiency of FE cathodes: $\boxed{v_{3E\_FE} = \frac{I_E}{P_{extr}} = \frac{1}{U_{extr}} \quad [V^{-1}].}$

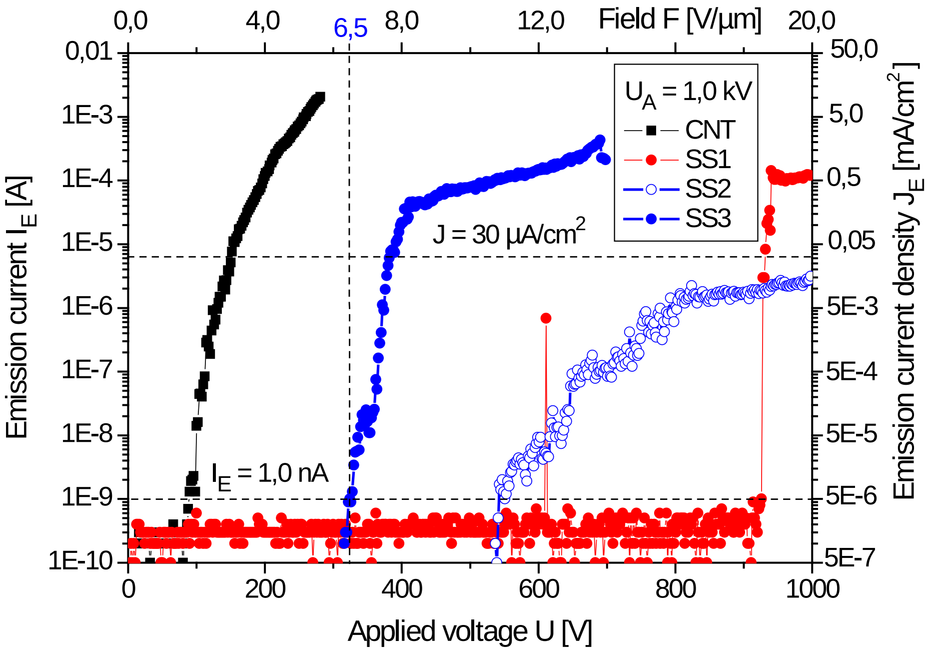
That means that $\frac{1}{U_{extr}}$ (and so U_{extr}!) is a directly measure of the emission efficiency of FE cathodes.

But also the inverse extraction voltage $\frac{1}{U_{extr}}\,[V^{-1}]$ is the **abscissa of the Fowler-Nordheim (FN) plot**!

The comparison with other emission mechanism is based on the calculation of a fictive extraction voltage: $\boxed{U^*_{extr} = P_{el}/I_E}$ The resulting **FN plot ordinate** is $\ln(I_E/(U^*_{extr})^2)$ for all electron emissions.

Fowler Nordheim (FN) plot

Standard use: measurement and calculation of field enhancement factor β



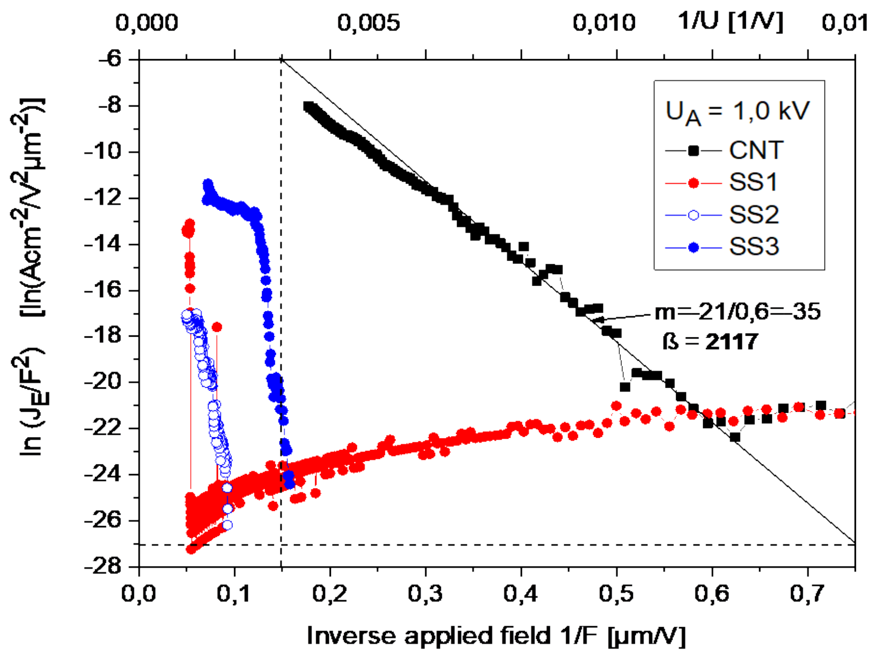


Fig. 1 Field-emission (FE) characterization of three stainless steel samples SS1, SS2 and SS3, and CNT Buckypaper with the same area for comparing [1]. For sample SS3 the current density value J = 30 µA/cm² lies significantly below the saturation characteristic.

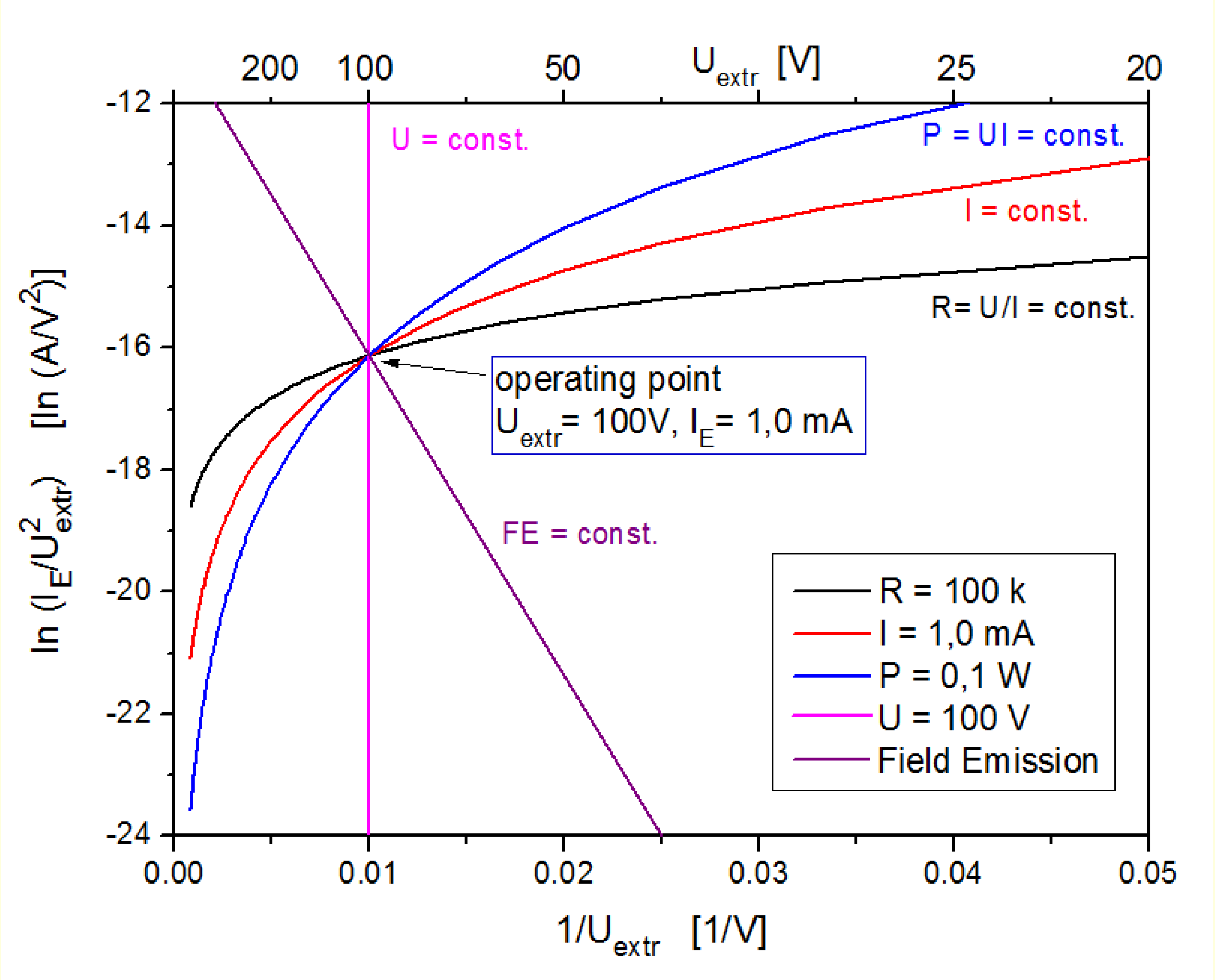
Fig. 2 Fowler-Nordheim plot (FN plot) of FE measurements in Fig. 1. and with graphic presentation of slope m for β_{CNT} calculation [2]:

$$\beta_{CNT} = - (B \cdot \Phi^{3/2}) / m$$

[1] W. Knapp, D. Schleussner, "Field-emission characteristics of carbon buckypaper", JVST B 21 557-561 (2003).

[2] W. Knapp "Field-emission investigations of micro-structured stainless steel 1.4301 (ASTM 304) for extended use of vacuum components as very large FE cathode arrays." Poster, IVNC 2017.

Fowler Nordheim (FN) plot

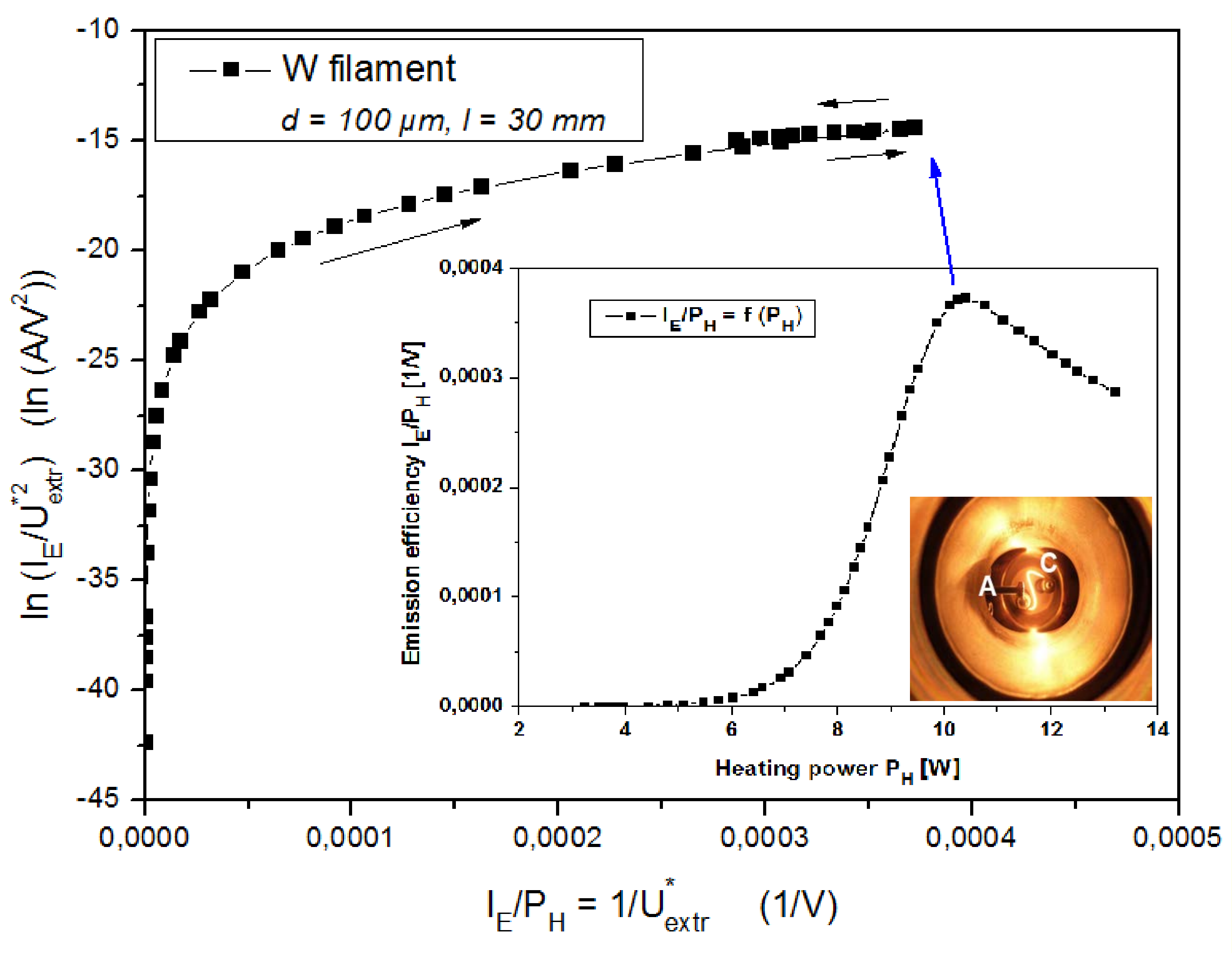


FN plot with constant characteristics based on an operating point

Knapp, Wolfram: „Using Fowler-Nordheim plot for the comparison of electron emission efficiency of field (FE) and thermal emission (TE) cathodes. 7th ITG International Vacuum Electronics Workshop (IVEW) 2020 and 13th International Vacuum Electron Sources Conference (IV*e*SC) 2020 , planned for Physikzentrum Bad Honnef (PBH), Germany (virtual conference/Thales AVS France), May 26-29, 2020, Lecture IVEW23, May 26, 2020, 3:30 pm MESZ.

Fowler Nordheim (FN) plot

1^{st} FN plot with hot filament characteristic



FN plot of HF cathode with increasing emission efficiency, cf. inside $I_E/P_{Heat} = f(P_{Heat})$

Knapp, Wolfram: „Using Fowler-Nordheim plot for the comparison of electron emission efficiency of field (FE) and thermal emission (TE) cathodes. 7th ITG International Vacuum Electronics Workshop (IVEW) 2020 and 13th International Vacuum Electron Sources Conference (IVeSC) 2020 , planned for Physikzentrum Bad Honnef (PBH), Germany (virtual conference/Thales AVS France), May 26-29, 2020, Lecture IVEW23, May 26, 2020, 3:30 pm MESZ.

Comparison of field (FE) and thermal (TE) cathodes

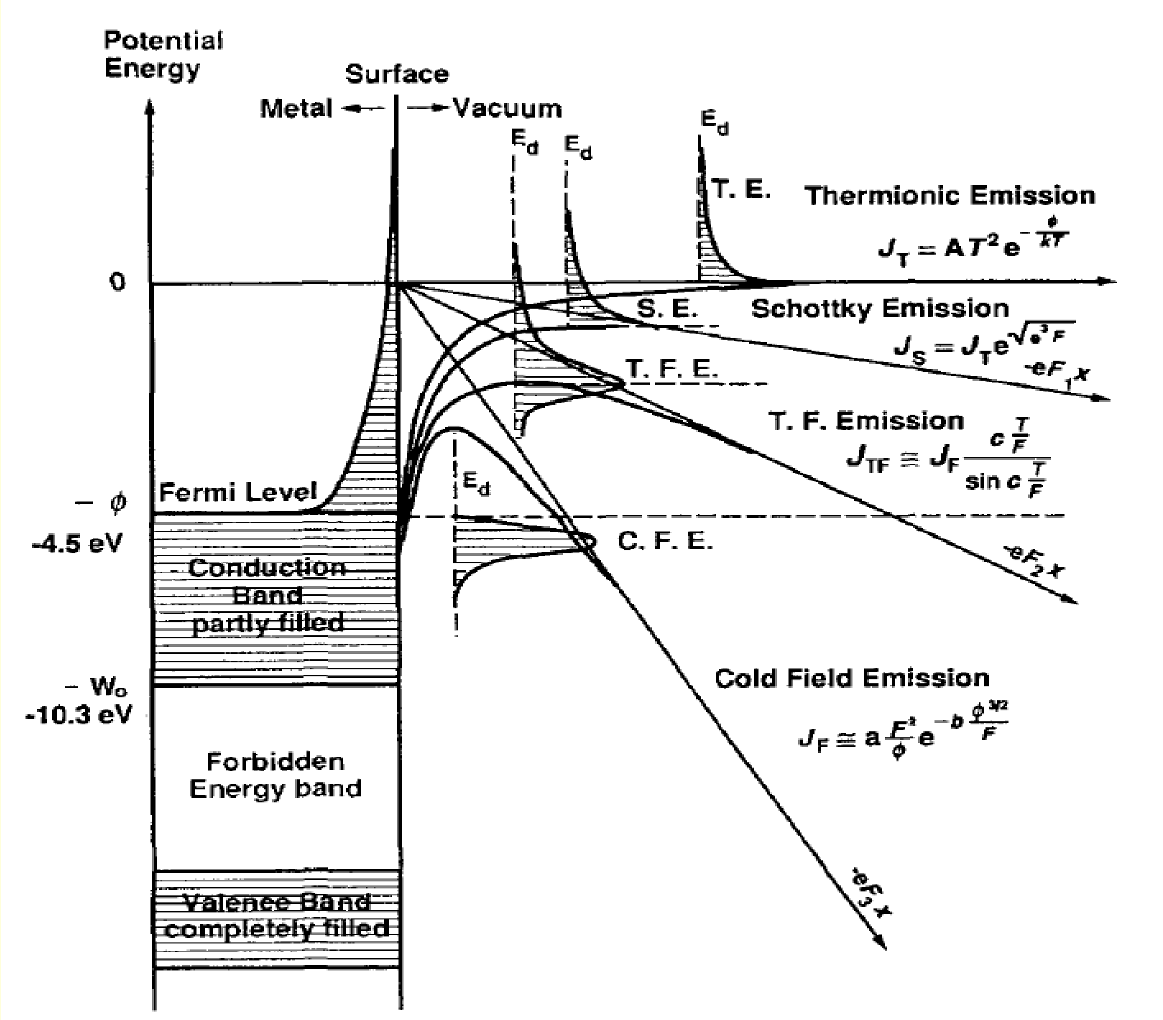
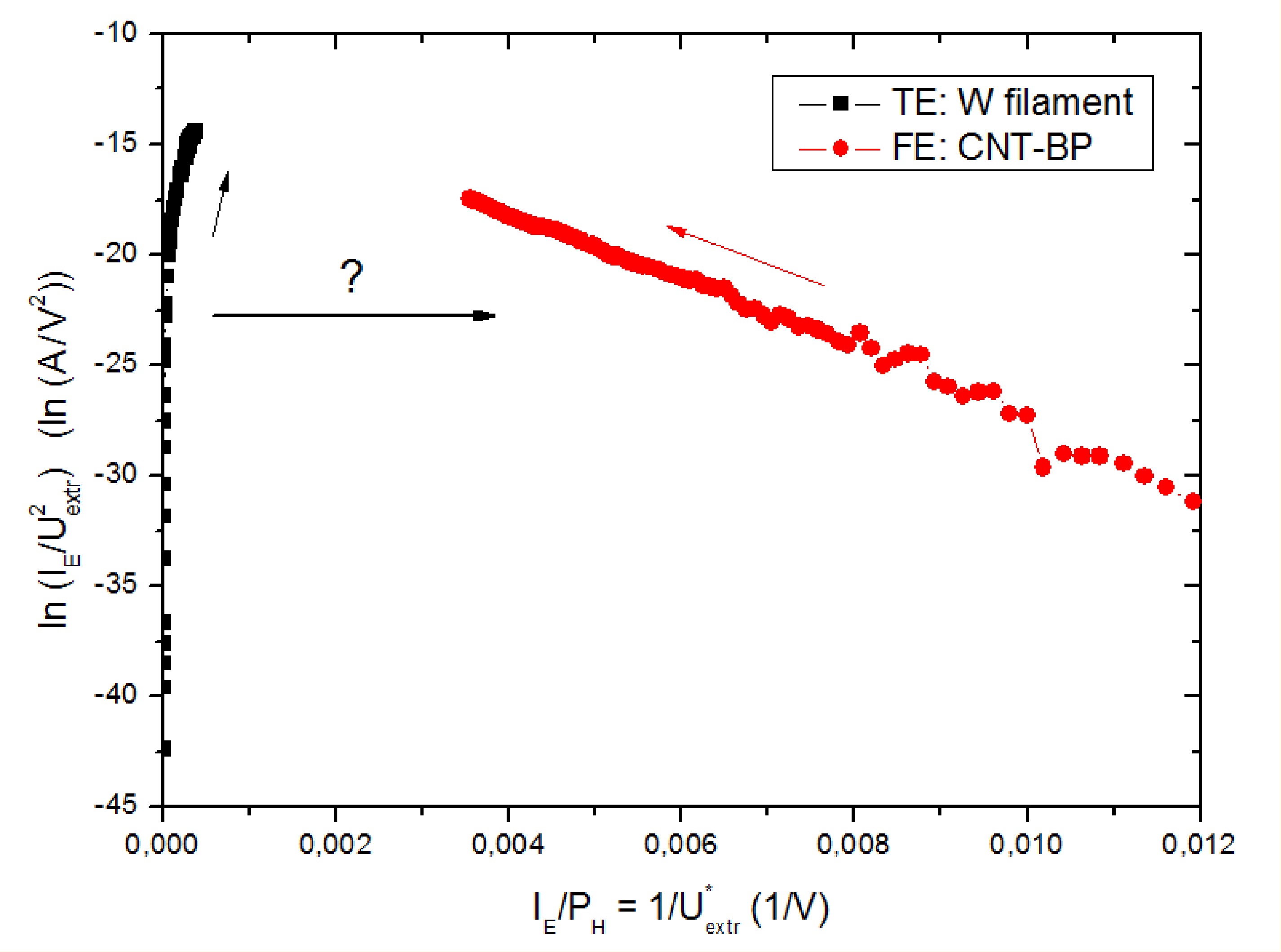


Fig. 1. Electron emission processes.

Lit.: F. Charbonnier: Developing and using the field emitter as a high intensity electron source, Applied Surface Science 94/95 (1996), p. 26-43.

Comparison of field (FE) and thermal (TE) cathodes



FN plot comparision of field (CNT-FE) and thermal TE cathodes (HF of BAG)

Knapp, Wolfram: „Using Fowler-Nordheim plot for the comparison of electron emission efficiency of field (FE) and thermal emission (TE) cathodes. 7th ITG International Vacuum Electronics Workshop (IVEW) 2020 and 13th International Vacuum Electron Sources Conference (IV*e*SC) 2020 , planned for Physikzentrum Bad Honnef (PBH), Germany (virtual conference/Thales AVS France), May 26-29, 2020, Lecture IVEW23, May 26, 2020, 3:30 pm MESZ.

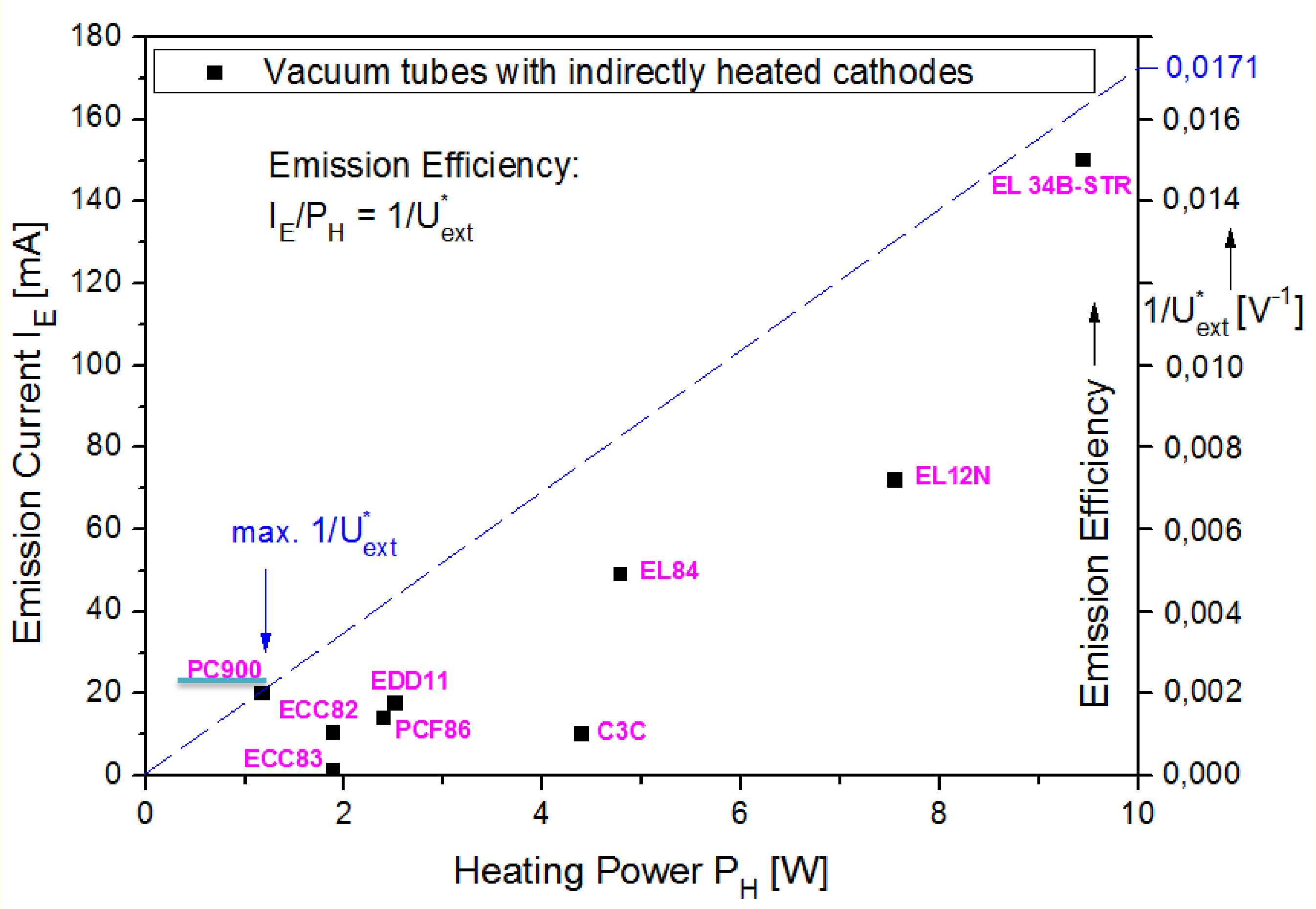
Comparison of field (FE) and thermal (TE) cathodes

Vacuum tubes with indirectly heated cathodes

Lit.: [1] Amplifier tubes, catalogue BEST OF reichelt 8/2019, S.18/19 (or: www.reichelt.de)

Comparison of field (FE) and thermal (TE) cathodes

Vacuum tubes with indirectly heated cathodes

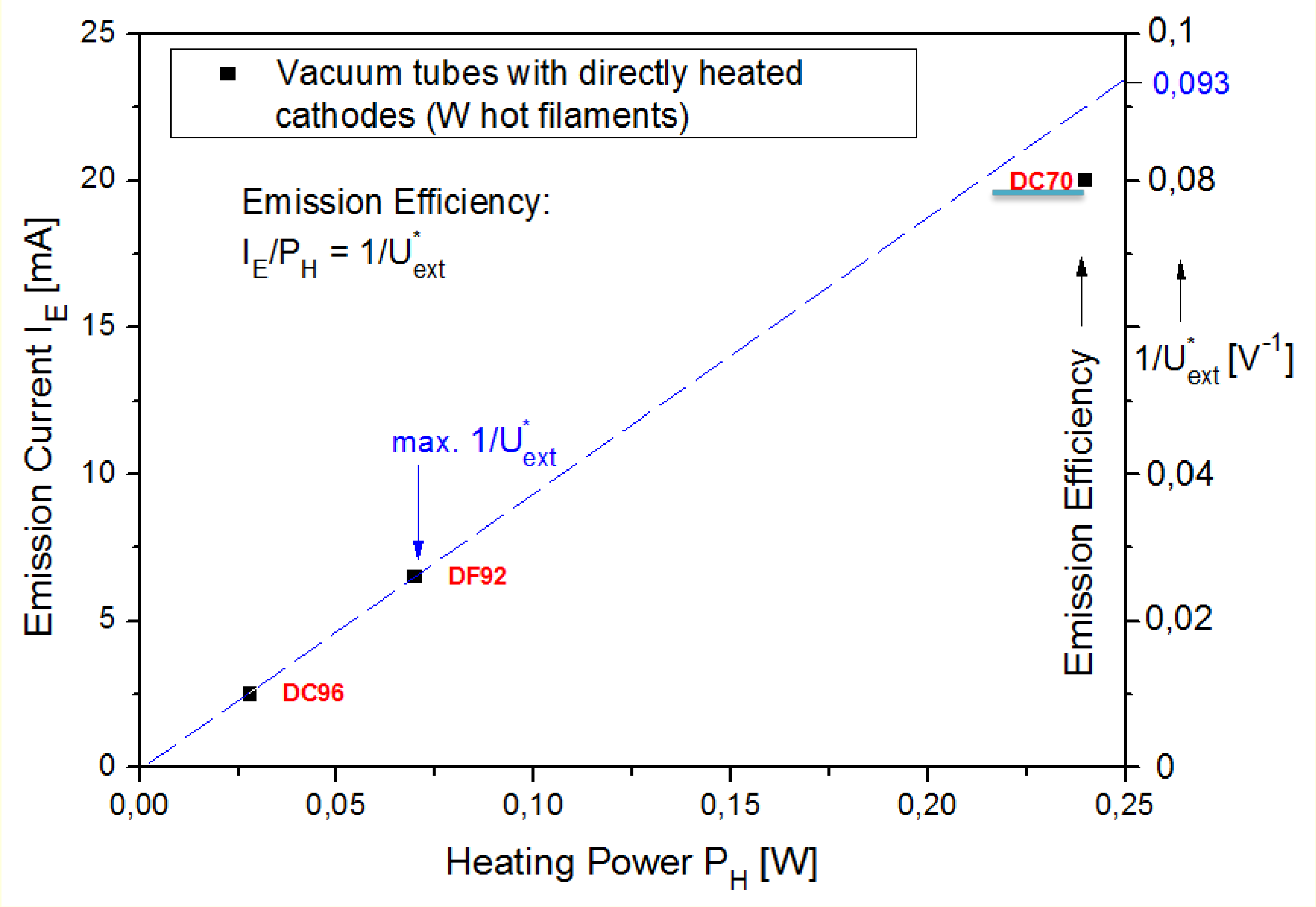


Selected vacuum tube: **PC900** (single hot filament, I_H = 300 mA, I_E = 20 mA) [1]

Lit.: [1] Amplifier tubes, catalogue BEST OF reichelt 8/2019, S.18/19 (or: www.reichelt.de)

Comparison of field (FE) and thermal (TE) cathodes

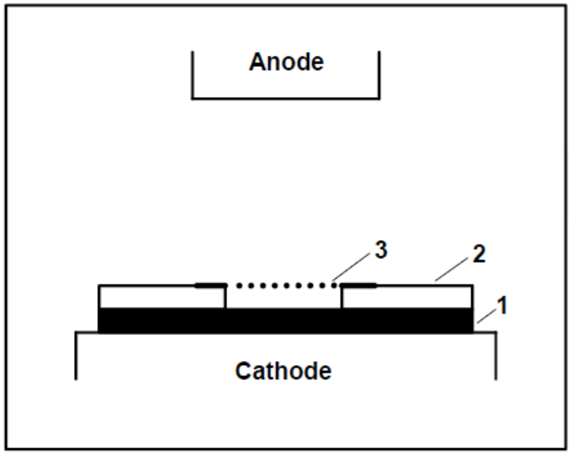
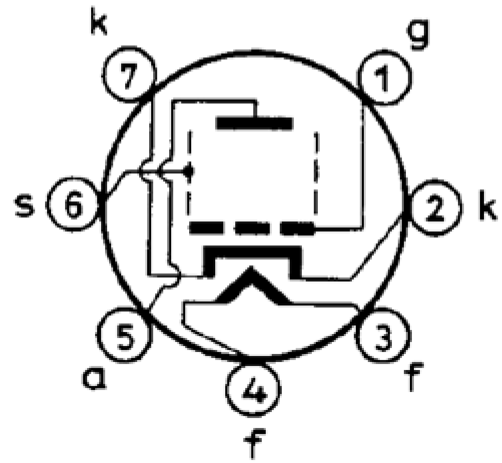
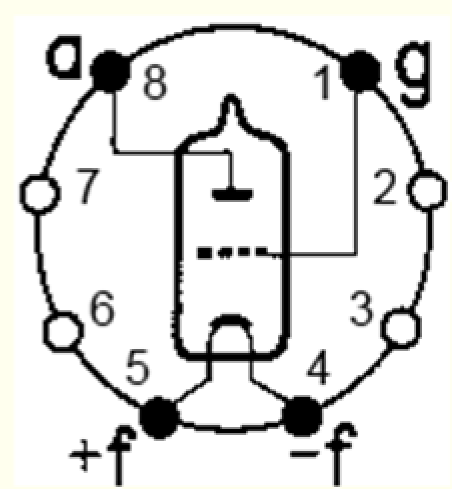
Vacuum tubes with directly heated cathodes



Selected vacuum tube: **DC70** (single hot filament, I_H = 200 mA, I_E = 20 mA)

Lit.: [1] Amplifier tubes, catalogue BEST OF reichelt 8/2019, S.18/19 (or: www.reichelt.de)

Comparison of field (FE) and thermal (TE) cathodes

<table>
<tr><th>Electron source with FE cathode [1]</th><th>Cathode of triode PC900</th><th>Cathode of triode DC 70</th></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td>Legend:
1 – cold field-emitter cathode with CNT Buckypaper
2 – spacer, 3 – micro grid</td><td>Legend:
f – hot filament, k – cathode, indirectly heated
g – grid, s – shield, a – anode</td><td>Legend:
f – hot filament cathode, directly heated
g – grid, a – anode</td></tr>
</table>

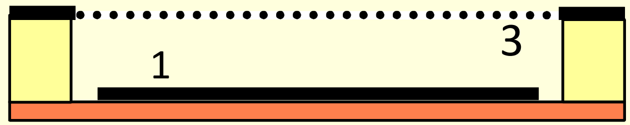
Schemes of triodes with compared FE and TE cathodes (U_{anode} = 100 V)

Ref.: [1] W. Knapp, D. Schleußner: Special features of electron sources with CNT field emitter and micro grid. Applied Surface Science **251** (2005) 164–169.

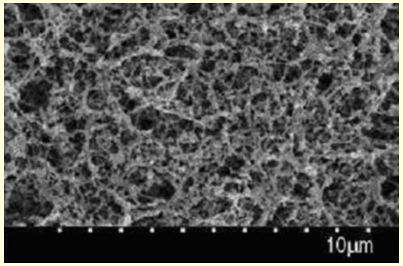
Comparison of field (FE) and thermal (TE) cathodes

Electron source with CNT field emission cathode for I_E = 1 mA (DC)
(reference source of Knapptron GmbH)

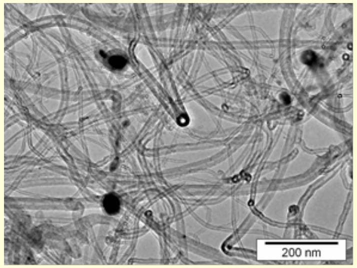
Layout of electron source :



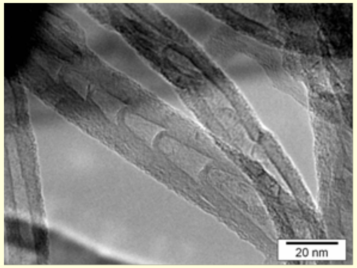
1. CNT field emission cathode with MWCNT [1]



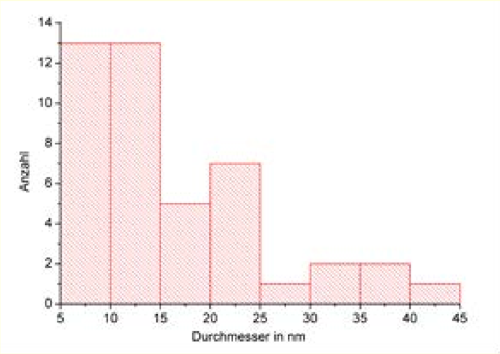
SEM image [10 µm]



TEM image 1 [200 nm]

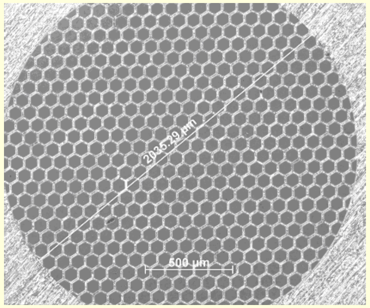


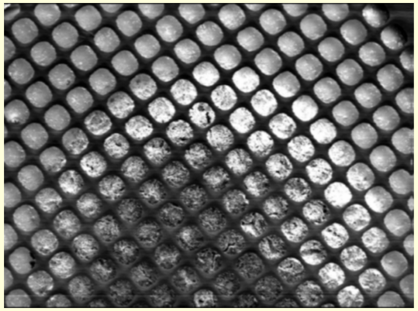
TEM image2 [20 nm]



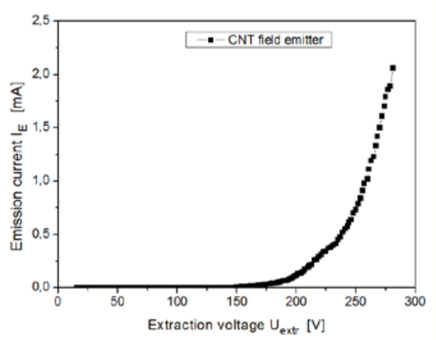
Distribution of d_{MWCNT} (TEM measurement)

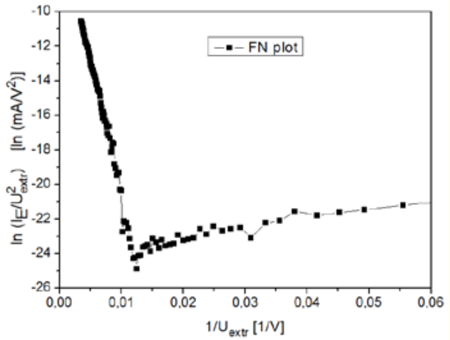
3. Micro grid [2]





Examples of micro grid





Electron field emission: I = f(U) and Fowler Nordheim (FN) plot

Lit.: [1] W. Knapp, D. Schleussner: Field-emission characteristics of carbon buckypaper. J. Vac. Sci. Technol. B 21 (2003) 1, 557 – 561.
[2] W. Knapp, D. Schleußner: Special features of electron sources with CNT field emitter and micro grid. Applied Surface Science 251 (2005) 164–169.

Comparison of field (FE) and thermal (TE) cathodes

Vacuum tube **PC900** (triode) with indirectly heated oxid cathode

PC900

V.H.F. TRIODE

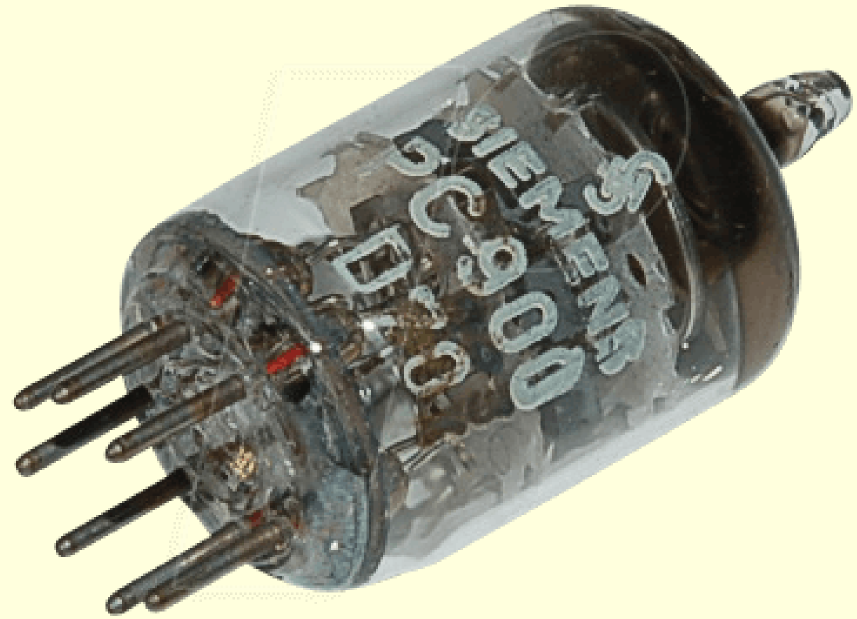
Triode intended for use as R.F. amplifier in V.H.F. television receivers.

QUICK REFERENCE DATA				
Cathode current	I_k	max.	20	mA
Transconductance	S		20	mA/V
Amplification factor	μ		84	

HEATING: Indirect by A.C. or D.C.; series supply

Heater current	I_f	300	mA
Heater voltage	V_f	3.9	V

P_f = 1,17 W



P_f = 1,17 W   and I_k = 20 mA

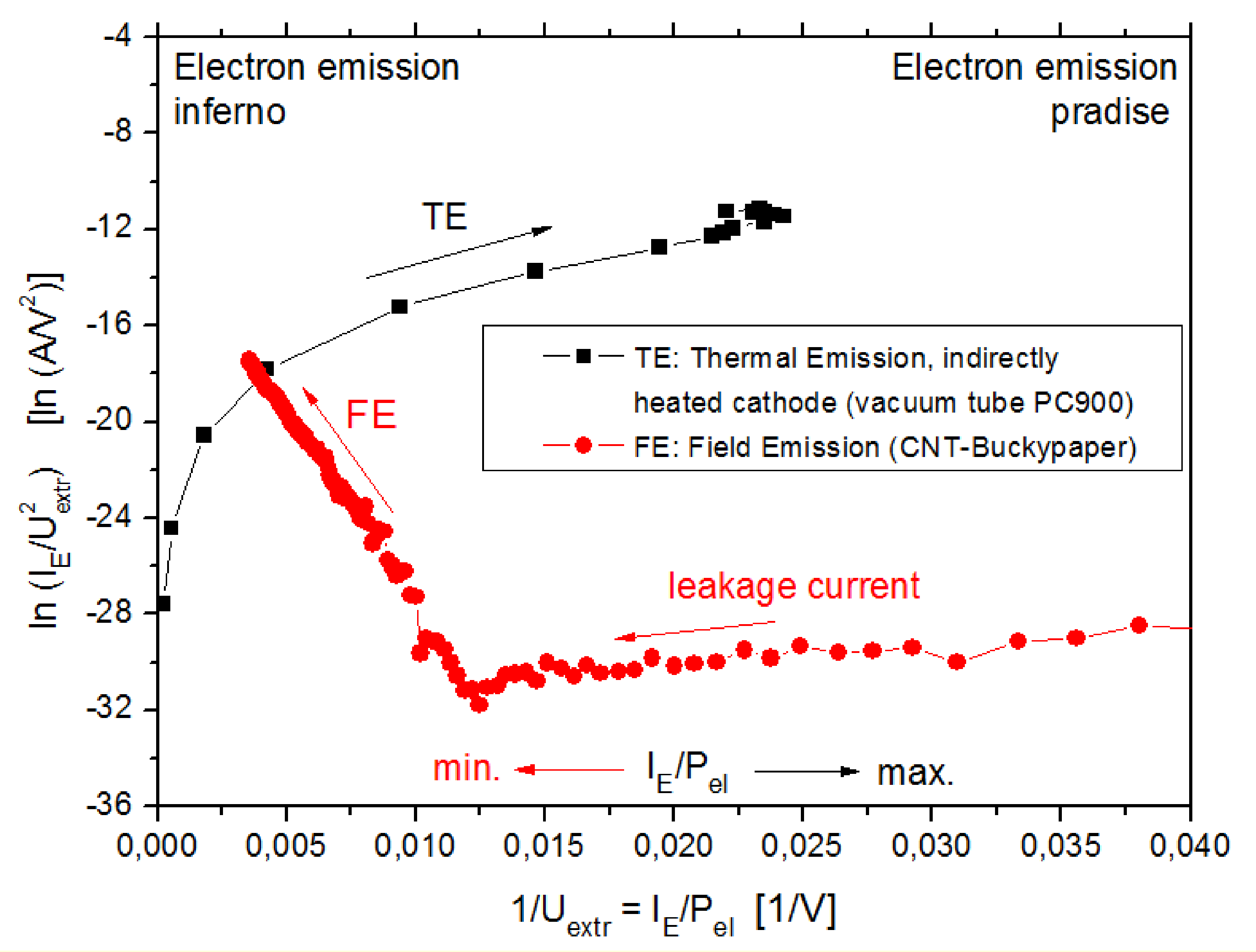
Emission efficiency: I_k/P_f = 0,0171

Fictive voltage: U^*_{extr} = 58,5 V

Actual price: 1,80 € (Germany)

Lit.: **Data sheet PC900**: https://cdn-reichelt.de/documents/datenblatt/A300/PC900_DB-EN.pdf (Januar 1970)

Comparison of field (FE) and thermal (TE) cathodes



FN plot comparision of field (CNT-FE) and thermal TE cathodes (indirectly heated)

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Comparison of TE and FE Electron Emission Efficiency
Different trends in dependence on physical parameters *T* or *U*

TE: The emission in temperature-limited regime is described by the **Richardson–Dushman equation:**

$$J_0 = A_R T^2 e^{-\Phi/kT} \qquad (1),$$

where J_0 [Acm^{-2}] is the current density at zero-field, A_R [$Acm^{-2}K^{-2}$] is the Richardson constant, in the ideal case equal to the thermionic constant 120.4 [$Acm^{-2}K^{-2}$], Φ [eV] is the electronic work function, k [eV/K] is the Boltzmann constant and T[K] is the true cathode temperature. The emission efficiency is:

$$v_{E3\_TE} = \frac{I_E}{P_{el}} \cong \frac{I_E}{P_{heat}} \cong \frac{A_C \cdot J_0}{Q(T)} = \frac{A_C[A_R \cdot \boldsymbol{T^2} e^{-\Phi/k\boldsymbol{T}}]}{m \cdot c \cdot \boldsymbol{T}(1 - T_a/\boldsymbol{T})} \approx \frac{\sim T^2}{\sim T} \sim T \quad \rightarrow \quad \boxed{\frac{\boldsymbol{I_E}}{\boldsymbol{P_{el}}} \sim \boldsymbol{T}} \qquad (2).$$

It means in effect: The emission efficiency is increasing with higher cathode temperature ***T*** (for higher I_E). In this regime **TE has a positive trend.**

FE: The field emission **FE has a negative trend** (cf. FN plot): $$v_{3E\_FE} = \boxed{\frac{\boldsymbol{I_E}}{\boldsymbol{P_{el}}} = \frac{\boldsymbol{1}}{\boldsymbol{U_{extr}}}} \qquad (3).$$

The emission efficiency is decreasing with higher extraction voltage $\boldsymbol{U_{extr}}$ and so for higher emission current $\boldsymbol{I_E}$, cf. FN equation: $J_E = \frac{I_E}{A_C} = A \frac{E_{loc}^2}{\Phi} e^{-B \cdot \Phi^{1,5}/E_{loc}}$ (4), with $E_{loc} = \gamma \cdot E_{extr} = \gamma \frac{U_{extr}}{d}$ (5).

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Comparison of field (FE) and thermal (TE) cathodes

Vacuum tube **DC 70** (triode) with directly heated hot filament cathode

SUBMINIATURE U.H.F. TRIODE DC 70

The DC 70 is a directly heated triode intended for purposes of transmitting and receiving at ultra high frequencies. It can for instance be used as an oscillator, amplifier, super-regenerative detector or mixer in walkie-talky equipment, balloon sondes, Citizens Radio or professional equipment, etc. When the tube is used as an oscillator, the output obtainable at 500 Mc/s ($\lambda = 60$ cm) is about 450 mW.

The filament voltage of the DC 70 is 1.25 V at a current of 0.2 A. Being a directly heated tube, its mutual conductance is high (3.4 mA/V at an anode current of 12 mA); the amplification factor amounts to 14.

The DC 70 is provided with leads which pass through the base and are soldered directly to the wiring of the circuit. For this reason no tube socket is used.

The small dimensions and battery operation make the DC 70 specially suitable for use in portable equipment. The tube can be mounted in all positions.

OPERATING CHARACTERISTICS AS AN OSCILLATOR AT

Anode voltage	V_a	=	150 V
Cathode current	I_k	=	20 mA

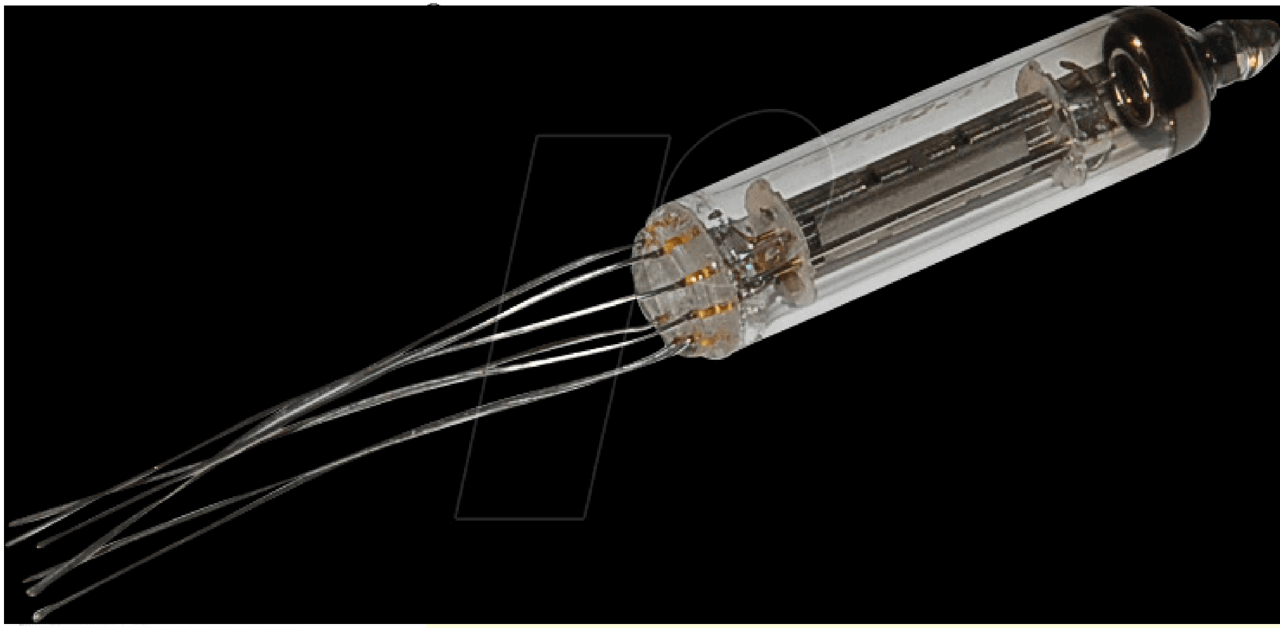


Fig. 1. Photograph of the DC 70 (actual size).

TECHNICAL DATE

FILAMENT DATA

Heating: direct by battery

Filament voltage	V_f	=	1.25 V
Filament current	I_f	=	0.20 A

P_f = 0,25 W

P_f = 0,25 W and I_k = 20 mA

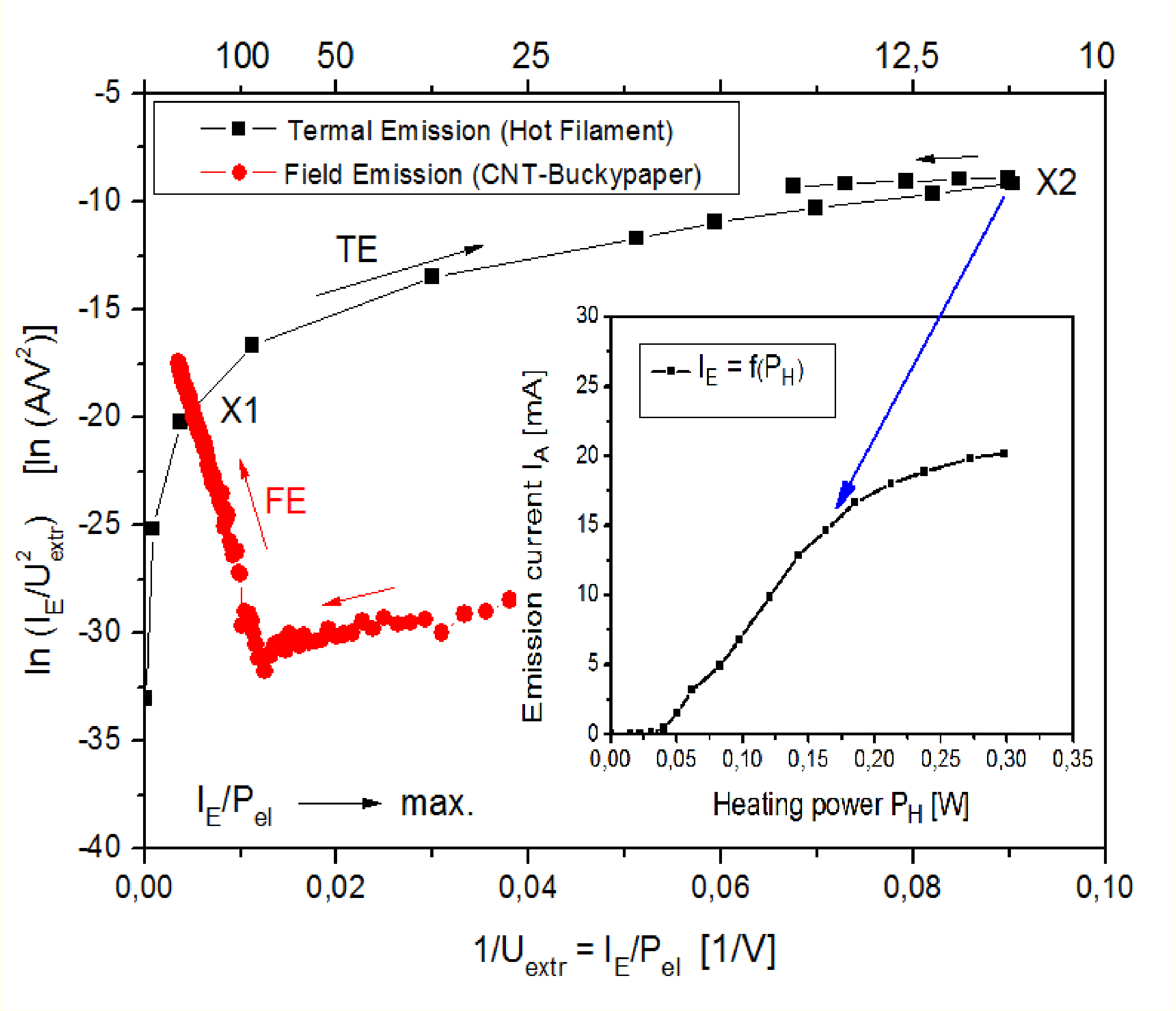
Emission efficiency: I_k/P_f = 0,08

Fictive voltage: U^*_{extr} = 12,5 V

Actual price: 5,35 € (Germany)

Lit.: **Data sheet DC70:** https://cdn-reichelt.de/documents/datenblatt/A300/DC70_DB-EN.pdf

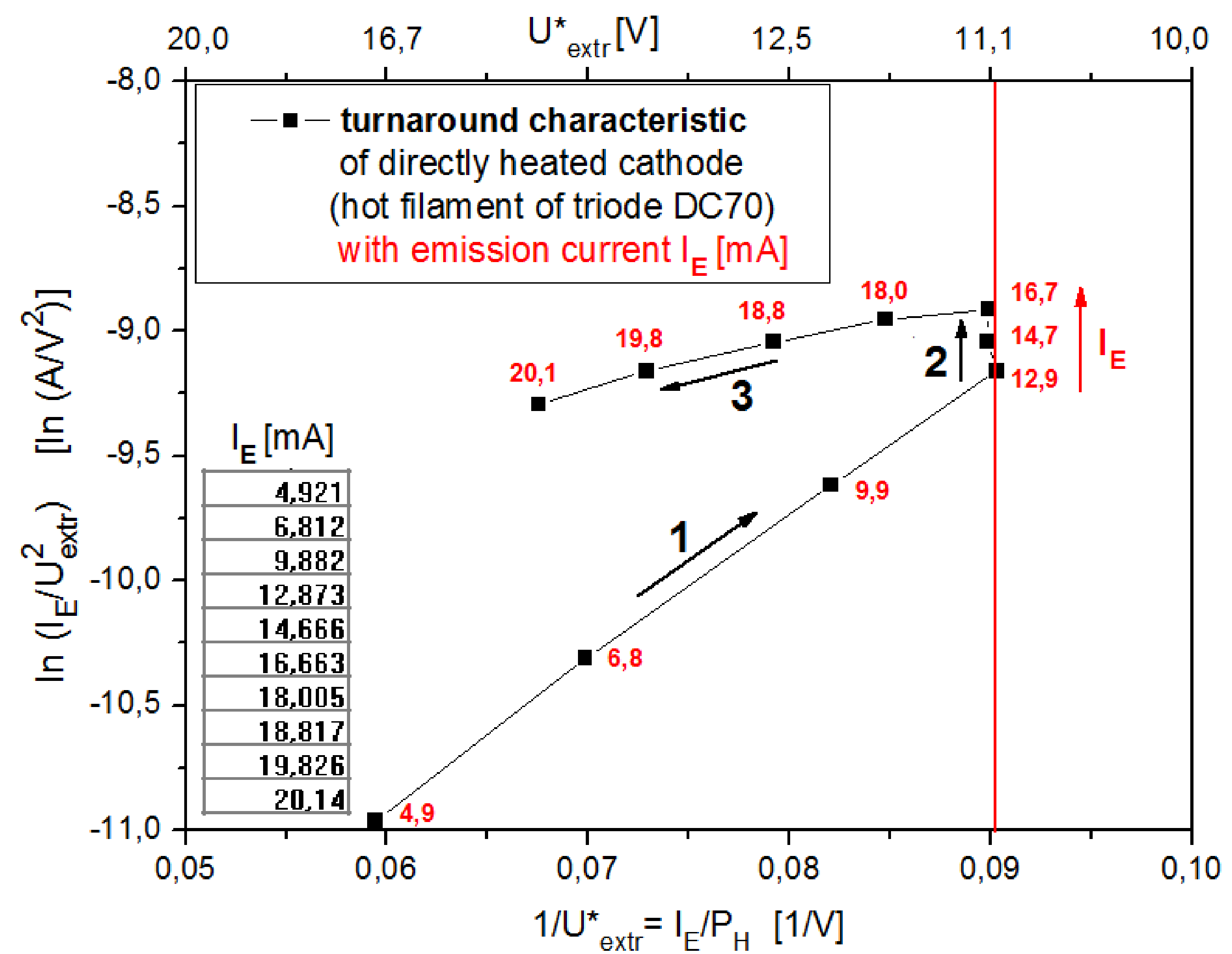
Comparison of field (FE) and thermal (TE) cathodes



FN plot comparision of field (CNT-FE) and thermal cathodes (hot filament of DC70)

Knapp, Wolfram: „Using Fowler-Nordheim plot for the comparison of electron emission efficiency of field (FE) and thermal emission (TE) cathodes. 7th ITG International Vacuum Electronics Workshop (IVEW) 2020 and 13th International Vacuum Electron Sources Conference (IV*e*SC) 2020 , planned for Physikzentrum Bad Honnef (PBH), Germany (virtual conference/Thales AVS France), May 26-29, 2020, Lecture IVEW23, May 26, 2020, 3:30 pm MESZ.

Comparison of field (FE) and thermal (TE) cathodes



TE emission efficiency

in the three curve sections:

1 $\frac{I_E}{P_{el}} \sim T$

Increasing with $T \uparrow$

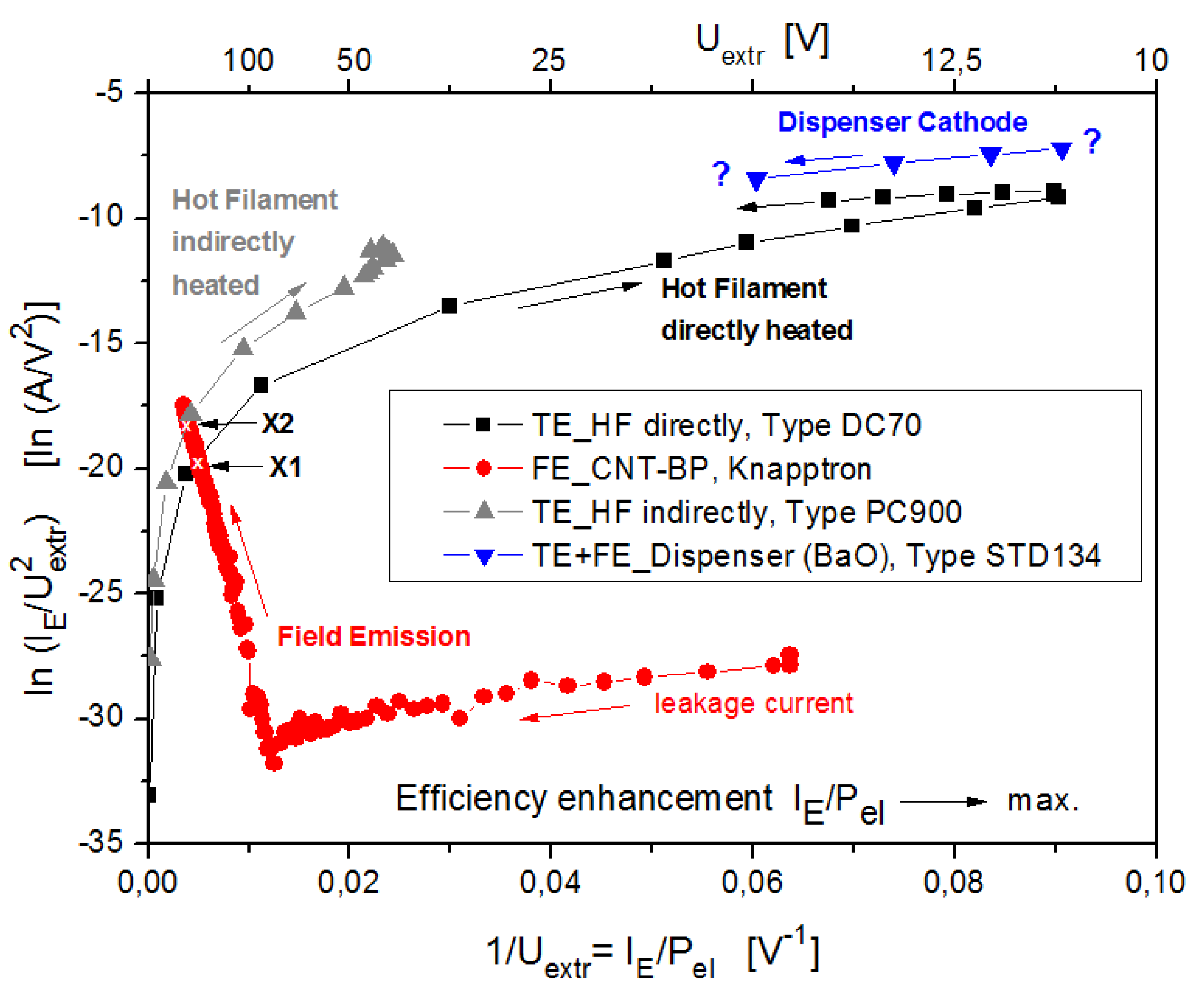
2 $\frac{I_E}{P_{el}} = \text{const.}$

3 $\frac{I_E}{P_{el}} = \frac{I_E}{P_{heat} + P_{loss}}$

Decreasing with $P_{loss} \uparrow$

FN plot analysis of TE (hot filament) with three significant curve sections and corresponding trends of electron emission efficiencies at higher temperature *T* (right).

Comparison of field (FE) and thermal (TE) cathodes



FN plot comparision of all investigated FE and TE cathodes

Knapp, Wolfram: „Using Fowler-Nordheim plot for the comparison of electron emission efficiency of field (FE) and thermal emission (TE) cathodes. 7th ITG International Vacuum Electronics Workshop (IVEW) 2020 and 13th International Vacuum Electron Sources Conference (IVeSC) 2020 , planned for Physikzentrum Bad Honnef (PBH), Germany (virtual conference/Thales AVS France), May 26-29, 2020, Lecture IVEW23, May 26, 2020, 3:30 pm MESZ.

Extended use of Fowler Nordheim plot Energetic evaluation of charge transfers

For novel investigations of the transition from field emission to stable plasma discharges an extended use of FN plot is proposed.

For this the inverse voltage (x-axis of the FN plot) is extended with factor 1 (= I·t / I·t):

$\mathbf{1/U} = I \cdot t / U \cdot I \cdot t = \mathbf{Q/E}$ (1),

where U is the voltage, I the current, t the time,
Q the sum of all transported charges and
E the energy required for this.

The ordinate of Fowler-Nordheim plot will be unchanged.

The relation Q/E is a possible option for energetic evaluation.

The **operation compass** is important for basic orientation.

The direction Q/E maximum (red arrow or x-axis: 1/U) is the direction of energetically optimal charge transfer.

It can be seen in Fig. 1 and Fig. 2 (next slide) that the micro-arc discharge has
the highest charge transfer with the least energy input.

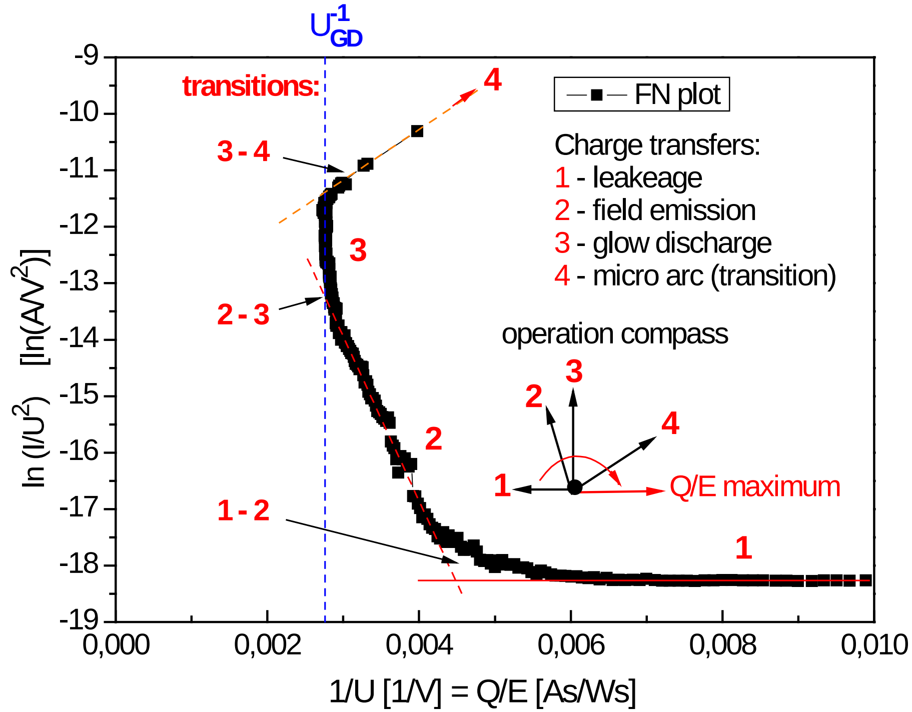


Fig. 1. FN plot of presented measurement SS3 [1] with smooth transitions. U_{GD} is the constant voltage of normal glow discharge.

[1] W. Knapp "Field-emission investigations of micro-structured stainless steel 1.4301 (ASTM 304) for extended use of vacuum components as very large FE cathode rrays." Poster, IVNC 2017.

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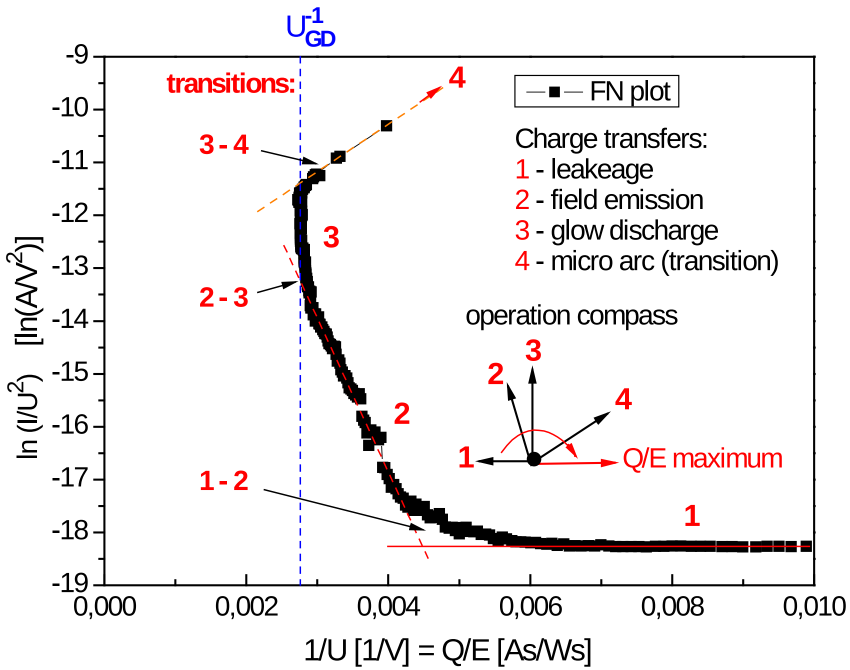
R_S = 100 kΩ, FE-cathode: stainless steel SS3 [1]



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[1] W. Knapp "Field-emission investigations of micro-structured stainless steel 1.4301 (ASTM 304) for extended use of vacuum components as very large FE cathode arrays." Poster, IVNC 2017.

[2] W. Knapp, C. Langer, C. Prommesberger, M. Lindner, R. Schreiner, "Investigations of the transition from field electron emission to stable plasma discharge in a micro electron source at vacuum pressure." Poster contribution, IVNC 2017.

R_S = 10 MΩ, FE-cathode: array of Si-tips [2]

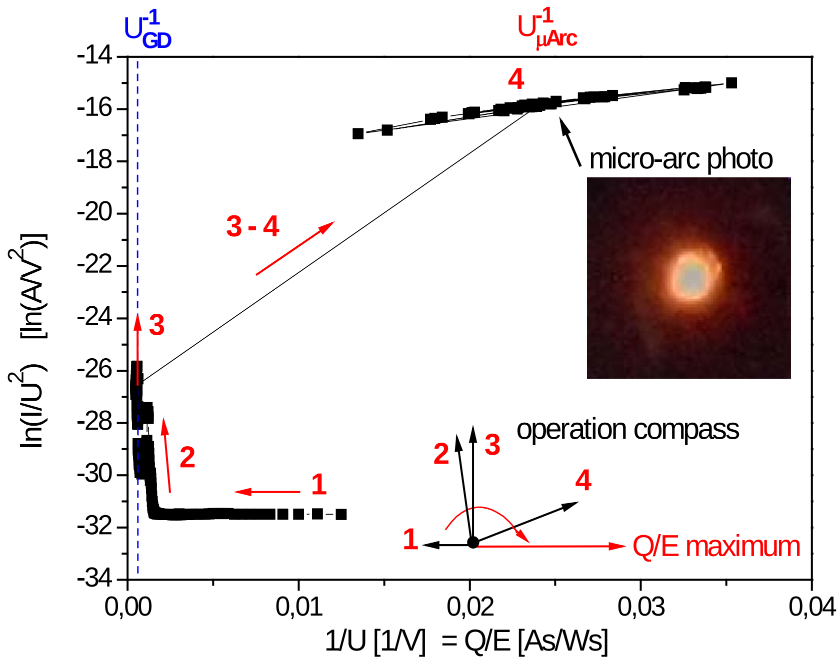

Fig. 2. FN plot for energetic evaluation of the transition from field electron emission to plasma discharges, as presented and detailed explained in [2, 3], with: **1** – leakage current, **2** – field emission, **3** – normal glow discharge, **3 - 4** – glow-to-arc transition, **4** – micro-arc discharge.

[3] D. Wenger, W. Knapp, B. Hensel, S. T. Tedde, „Transition of Electron Field Emission to Normal Glow Discharge", IEEE Transaction on Electron Devices 61 (11), p. 3864 (2014).

Conclusion and outlook

1. An extended use of FN plot is proposed for comparison of electron emission efficiency, in particular for field (FE) and thermal (TE) cathodes.
2. For this the inverse extraction voltage of field emission (x-axis of FN plot) is extended with factor I_E/I_E : $\frac{\mathbf{1}}{\boldsymbol{U_{extr}}} = \frac{\boldsymbol{I_E}}{\boldsymbol{P_{el}}}$.
3. The relation I_E/P_{el}, easy to measure (e.g. for TE is $P_{el} = P_{heat}$), is therefore a simple criterion reflecting the electron emission efficiency, independent of the kind of electron emission mechanism.
4. The comparison of different electron emission mechanism is based on the calculation of a fictive extraction voltage, e.g. for thermal emission: $\boldsymbol{U^*_{extr} = P_{heat}/I_E}$.
5. First comparison results of emission efficiency of FE and TE cathodes are:

- For higher current I_E: TE has an increasing and FE has a decreasing emission efficiency!
- The cross-over point *X* of characteristics has a current value e.g. of $I_{E\_X} = 117\ \mu A$.
- For $I_E > I_{E\_X}$ TE (hot filament cathode of vacuum tube DC70) becomes more efficient.
- For currents $I_E < I_{E\_X}$ FE cathodes (investigated with CNT emitter) are more efficient.
- FE requires more voltage U_{extr} for higher emission current I_E. With higher voltage a transition to plasma discharge are more likely to take action, if $I_E > 1.0\ mA$.

6. The influence of power loss on the emission efficiency must be investigated more!

Knapp, Wolfram: „Using Fowler-Nordheim plot for the comparison of electron emission efficiency of field (FE) and thermal emission (TE) cathodes. 7th ITG International Vacuum Electronics Workshop (IVEW) 2020 and 13th International Vacuum Electron Sources Conference (IV*e*SC) 2020 , planned for Physikzentrum Bad Honnef (PBH), Germany (virtual conference/Thales AVS France), May 26-29, 2020, Lecture IVEW23, May 26, 2020, 3:30 pm MESZ.

Thank you very much for your attention!

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